RANDOM LASER WITH COLD ATOMS:
EXTRACTING INFORMATION FROM
ATOMIC FLUORESCENCE

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Introduction

In a gain medium, when multiple scattering is present, this can provide a feedback, acting like the cavity of a canonical laser. There is a threshold on the sample size as predicted by Letokhov in 1968 [1]. This effect is called “random laser”.

In the past few decades many studies have been carried out in the random laser field. The experimental realizations has been done adding strongly scattering materials to laser dye [2] or grinding solid state lasers like semiconductor into powder [3].

In the cold atoms field many studies on multiple scattering of light inside the sample have been carried out [4, 5] and the possibility of adding gain in such an atomic sample has been experimentally investigated [6, 7].

At the “Institut Non Linéaire de Nice” (INLN, Sophia-Antipolis) a research project is in progress, which aims at combining gain and multiple scattering in order to realize a random laser with cold atoms (CAROL-ANR project). My work in this project has consisted in the experimental measurement of the random laser threshold for the Raman gain mechanism, and in some preliminary studies on the detection of the future random laser emission. Both these parts has involved the detection and the study of the fluorescence light from the atomic sample.

In the chapter 1 I will explain in more detail what is a random laser in the general case and which are its features. I will derive the Letokhov threshold formula and I will expose the present knowledge about the random laser properties. Subsequently I will explain the cold atoms project and which are the differences and the challenging problems compared to the other random laser realizations.

In the chapter 2 I will explain the theory of laser cooling and trapping of atoms and I will present the experimental apparatus I worked on. In the chapter 3 I will present some possible gain mechanisms with cold atoms, in particular the Raman gain.

In the chapter 4 I will present my work about the threshold prediction with the Raman gain. I will explain my experimental work to measure the transmission and the scattering rate from a laser probe by the cold atoms sample. Then I will present how I have extracted the characteristic lengths of the sample from the previous measurements. Finally I will present our
numerical evaluation of the scattering rate starting from the transmission measurement.

In the last chapter (ch. 5) I will present some detection technique we anticipate to be useful for the detection of the random laser emission. I will explain the optical apparatus we have built in order to detect a signal in one spatial coherence area (a the single speckle grain). I will present the study I have done on two different detectors, and I will conclude exposing the features that an appropriate random laser detector must have.
Chapter 1

A Random Laser with cold atoms?

1.1 Random Laser

A laser is usually constructed from two basic elements: a material that provides optical gain through stimulated emission, and an optical cavity that partially traps the light. Laser emission occurs when the total gain in the medium is larger than the losses of the cavity. The cavity determines the modes of a laser: frequency, direction and spatial shape. In a random laser, multiple scattering plays the role of the cavity.

A scattering event occurs when a photon change direction after an interaction with an atom. Multiple scattering is a well-known phenomenon that occurs almost in all optical material that appear opaque, such as clouds, milk, white paints, fog. The photons do a lot of scattering events before escaping from the material. The light that propagates in the medium makes a random walk, as the brownian motion of particle suspended in a liquid. In this way the light makes longer paths. In a conventional laser multiple scattering has to be avoided because it adds some unwanted losses. But in an optical material it is hard to avoid completely the diffusion process due to material defects. In a random laser the more scattering we have the better it is.

To realize a random laser it is important to have an optically thick sample: the mean free path $\ell_{sc}$ should be at least smaller than the sample thickness $L$, otherwise the sample becomes transparent; and there should be enough gain, provided by stimulated emission, to reach the threshold.

In such an amplifying random medium, light waves are multiply scattered and amplified.

The relevant length scales that describe the scattering process are the scattering mean free path $\ell_{sc}$ and the transport mean free path $\ell_t$. The former is defined as the average distance that light travels between two
consecutive scattering events, and the latter is the average distance a wave travels before its direction of propagation is randomized. These two lengths scales are related by $\ell_t = \frac{\ell_{sc}}{1 - \langle \cos \theta \rangle}$, where $\langle \cos \theta \rangle$ is the average cosine of the scattering angle. For Rayleigh scattering we have $\langle \cos \theta \rangle = 0$ and so $\ell_t = \ell_{sc}$, while for Mie scattering we have $\langle \cos \theta \rangle \approx 0.5$ and so $\ell_t \approx 2\ell_{sc}$. There are three regime for light transport in a 3D random passive medium [8]:

- $\ell_t \geq L$, the ballistic regime;
- $L \gg \ell_t \gg \lambda$, the diffusive regime;
- $k_l \cdot \ell_{sc} \approx 1$ the localization regime ($k_l$ is the effective wavevector in the random medium).

Light amplification by stimulated emission is described by the gain length $\ell_g$ and the amplification length $\ell_{amp}$. The gain length $\ell_g$ is the path length over which the intensity is amplified by a factor $e$, and $\ell_{amp}$ is the average distance between the beginning and the ending point for path of length $\ell_g$. In a homogeneous medium without scattering, light travels in a straight line, thus $\ell_{amp} = \ell_g$. In the diffusive regime, $\ell_{amp} = \sqrt{D t}$, where $D$ is the diffusion coefficient, $t = \ell_g / v$ and $v$ is the speed of light. In a three-dimensional system, $D = v \ell_t / 3$, thus $\ell_{amp} = \sqrt{\frac{2\ell_t}{3}}$. Such length must be compared to the linear dimension of the sample $L$ in order to predict a threshold.

### 1.1.1 Coherent and Incoherent feedback

In a conventional laser, the cavity provides a coherent feedback. That means that the reflection by a mirror does not destroy the phase of the field and the phase relations between two points of the wavefront, and so there are interference effects that determine the resonant frequencies and the spatial modes of the cavity. These frequencies, that spent a lot of time in the cavity, are amplified by stimulated emission. When the optical gain balances the loss of a resonant mode, lasing oscillation occurs in this mode. The threshold condition is

$$R_1 R_2 e^{2g L_c} = 1$$

where $R_1$ and $R_2$ are the reflectivities of the two mirrors, $L_c$ is the cavity length, and $g$ is the linear gain coefficient.

If we replace one mirror of the Fabry-Perot cavity with a scattering surface we obtain a laser cavity that provides non-resonant feedback. Light in the cavity suffers multiple scattering; its direction is changed every time it is scattered. In a single scattering the phase is not destroyed but every point of the surface scatters with a different random phase. The phase relations between different points of the wavefront are destroyed. The spatial resonances for the electromagnetic field are absent in such a cavity. The lifetime
of light in the cavity is not sensitive to its frequency in the sense that such a cavity does not select any frequencies. In the case of cold atoms it is different because the scattering process depends on the transition frequency of the atoms and so it depends on frequency but not like cavity modes. The feedback in such a laser is used merely to return part of the photons to the gain medium; it is energy or intensity feedback [8]. The incoherent feedback could also be interpreted in term of modes. When one of the mirror is replaced by a scattering surface, escape of emission from the cavity by scattering becomes the predominant loss mechanism for all modes. Instead of individual high-Q resonances, there appear a large number of low-Q resonances which spectrally overlap and form a continuous spectrum. This corresponds to the occurrence of non-resonant feedback. The absence of resonant feedback means that the cavity spectrum tends to be continuous. The only resonant element left in this kind of laser is the amplification line of the active medium. With an increase of pumping intensity, the emission spectrum narrows continuously towards the center of the amplification line. However the process of narrowing is much slower than in an ordinary laser.

![Image of canonical cavity laser and random laser idea](image_url)

**Figure 1.1:** a) Canonical cavity laser; b) Random laser idea. Figure from [9]

In a multiple scattering medium with gain the feedback could be either coherent or incoherent. There is a coherent feedback when the scattering is so high that the creation of closed loop in light paths is not negligible. And this closed loop provides interference effects and mode selection. This situation is reached when $\ell_{sc} \sim \lambda$. Another case of coherent feedback is that of boundary internal reflection that provides a sort of cavity. On the other way there is an incoherent feedback when after a scattering the photon is sent back in the gain medium, and the probability of returning in the original
position is negligible. In this case there are not any interference effects and the light makes a sort of random amplified walk.

Coherent feedback is not required to obtain coherent random lasing. The reason is that, like in a regular laser, it is not the cavity itself that is essential for obtaining coherent laser emission. To understand this better, first- and second-order coherence should be distinguished between. First-order coherence is a measure of fluctuations of the field, whereas second-order coherence accounts for fluctuations of the intensity. For a source of sufficiently narrow bandwidth, the first-order coherence is automatically high. Therefore, any mechanism that selects a specific narrow wavelength band (for example, a bandpass filter) creates first-order coherence. Second-order coherence is more difficult to obtain owing the tendency of photons to “bunch”, which creates large intensity fluctuations. In a laser, second-order coherence is obtained by saturation of the gain. This non linear effect limits the fluctuations of the intensity and thereby increases second-order coherence. If light is first- and second-order coherent, the emission can be called “coherent”. A laser cavity creates feedback and thereby forms a convenient mechanism that automatically leads to the gain saturation required for second-order coherent light. However, there are other situations in which gain saturation creates light that has second-order coherence. An example is that of the amplification of spontaneously emitted photons by stimulated emission. If the gain is large, the intensity will grow such that it depletes the gain medium completely. This will suppress the fluctuations of the intensity and thereby give rise to second-order coherence, or in other words to the characteristic “poissonian” photon statistics that characterize the coherent emission of a laser source. This also explains how a random laser can exhibit coherent emission, independently of the degree of localization of the modes and the amount of “coherent” feedback.

1.1.2 Letokhov photonic bomb

When the photon mean free path $\ell_{sc}$ is much smaller than the dimension of the scattering medium but much larger than the optical wavelength $\lambda$, the motion of photons is diffusive. In 1968 Letokhov [1] solved the diffusion equation for the photon energy density $W(\vec{r}, t)$ in the presence of a uniform and linear gain:

$$\frac{\partial W(\vec{r}, t)}{\partial t} = D \nabla^2 W(\vec{r}, t) + \frac{v}{\ell_g} W(\vec{r}, t)$$

(1.1)

where $v$ is the transport velocity of light inside the scattering medium, $\ell_g$ is the gain length and $D$ is the diffusion coefficient given by

$$D = \frac{v \ell_t}{3}.$$  

(1.2)
The general solution of equation 1.1 can be written
\[ W(\vec{r}, t) = \sum_n a_n \Psi_n(\vec{r}) e^{-(DB_n^2 - v/\ell_g)t}, \] (1.3)
where \( \Psi_n(\vec{r}) \) and \( B_n \) are the eigenfunction and eigenvalues of the following equation:
\[ \nabla^2 \Psi_n(\vec{r}) + B_n^2 \Psi_n(\vec{r}) = 0 \] (1.4)
with the boundary condition that \( \Psi_n = 0 \) at a distance \( z_\ell \) beyond the physical boundary of the scattering medium. \( z_\ell \) is the extrapolation length. Usually \( z_\ell \) is much smaller than the dimension of the scattering medium and can be neglected. Hence, the boundary condition becomes \( \Psi_n = 0 \) at the boundary of the random medium.

The solution of \( W(\vec{r}, t) \) in equation 1.3 changes from an exponential decay to an exponential increase in time when crossing the threshold
\[ DB_1^2 - \frac{v}{\ell_g} = 0, \] (1.5)
where \( B_1 \) is the lowest eigenvalue. If the scattering medium has a shape of a sphere of diameter \( L \), \( B_n = 2\pi n/L \), and the smallest eigenvalue is \( B_1 = 2\pi/L \). Substituting \( B_1 \) and \( D \) into equation 1.5, the threshold condition predicts a critical diameter
\[ L_{cr} = 2\pi \sqrt{\frac{\ell_g \ell_m}{3}}. \] (1.6)
In agreement with the fact that the total gain is proportional to the volume and the losses are proportional to the total surface, it exists a critical volume \( V_{cr} \sim L_{cr}^3 \) above which gain becomes larger than loss, and the intensity diverges exponentially with \( t \). This can be understood intuitively in terms of two characteristic length scales. One is the generation length \( L_{gen} \), which represents the average distance a photon travels before generating a second photon by stimulated emission. \( L_{gen} \) can be approximated by the gain length \( \ell_g \). The other is the mean path length \( L_{pat} \) a photon travels in the scattering medium before escaping through its boundary. \( L_{pat} = vL^2/D \). When \( V \geq V_{cr}, L_{pat} \geq L_{gen} \). This means on average every photon generates, at least, another photon before escaping the medium. This triggers a “chain reaction”, i.e. one photon generates two photons, two photons generates four photons, etc. Thus the photon number increases with time. Because this process of photon generation is analogous to the multiplication of neutrons in an atomic bomb, this device is sometimes called a photonic bomb. In reality light intensity does not diverge; there is no “explosion” because of gain saturation, as in a standard laser, and \( \ell_g \) increases.

If the gain and/or the scattering depend on the wavelength, this model also predicts that the emission spectrum narrows down above threshold with
a maximum intensity at the wavelength of maximum gain. This happens because the wavelength at maximum gain reaches the threshold before the other wavelengths, and furthermore it has an higher gain even when the others reach the threshold. In addition, relaxation oscillations as well as laser spiking can be found in such a diffusive model [10].

Several of these features have been observed in experiment (see sec. 1.1.4). The providing of gain and multiple scattering at the same time has been realized in different ways, mainly grinding semiconductor laser crystal into a fine powder or suspending microparticles in laser dye (the so called laser paint).

Although a simplified model of diffusion with gain, as originally discussed by Letokhov, is very powerful in predicting certain properties of a random laser, it also neglects some important aspects [11]. In particular, it neglects the fact that light rays in a random laser, while undergoing a random walk, are subject to interference effects. In the following section I will explain more deeply the importance of these interference effects.

### 1.1.3 Interference effects and multiple scattering

It’s important to note that if the incident radiation is weak the scattering process is elastic [12]. That means that only the direction change; it preserves the frequency of the incident radiation and the phase is well defined with respect to the incident field. The atoms act as classic dipoles oscillating under an external guiding field. It is important to notice that the polarization is randomized even in an elastic diffusion. The condition for elastic scattering process is that the saturation parameter $s$ of the transition is much less than 1. The saturation parameter is defined as

$$s = \frac{I}{I_{sat}} \frac{1}{1 + 4 \frac{\Delta^2}{\Gamma^2}} \quad (1.7)$$

where $I$ is the intensity of the incident radiation, $\Delta = \omega_L - \omega_0$ is the detuning of the incident radiation from the atomic resonance, $\Gamma$ is the linewidth of the transition and $I_{sat}$ is the saturation intensity of the atoms at resonance.

The elastic and inelastic scattering rate are respectively

$$S_{elastic} = \frac{\Gamma}{2} \frac{s}{(1 + s)^2} \quad (1.8)$$

$$S_{inelastic} = \frac{\Gamma}{2} \frac{s^2}{(1 + s)^2} \quad (1.9)$$

(See figure 1.2). And the total scattering rate is:

$$S = \frac{\Gamma}{2} \frac{s}{(1 + s)} = \frac{\Gamma}{2} \frac{I/I_{sat}}{1 + 4 \frac{\Delta^2}{\Gamma^2} + \frac{I}{I_{sat}}} \quad (1.10)$$
So $s \ll 1$ means that, if $\delta = 0$, $I \ll I_{\text{sat}}$, if $\delta \neq 0$ the condition on $I$ is not so strong.

In this condition, in which the scattered light has a well defined phase with respect to the incident field, the interference effects become important also after multiple scattering. This shows how interference is important.

It is the interference in the multiple scattering process that determines the spatial and spectral mode structure of a random laser. Interference in multiple scattering leads to a granular distribution of the intensity. The spatial profile of the modes is thus dominated by a speckle pattern with a gradually varying envelope; this is present in all kind of random laser. This means that interference effects cannot be neglected in all kind of random laser.

In most random materials, the intensity is spread throughout the sample and the modes are therefore called extended. In certain random materials, interference can lead to an effect called light localization, and this kind of modes are called localized. Light localization is the optical counterpart of Anderson localization of electrons. Owing to interference and thereby the multiple scattering process, the propagation of light waves comes practically to a halt in that case. This can be understood in terms of the formation of randomly shaped but closed loops. And so there are the formation of something like different cavities with different losses and frequency selections. Localization can only take place in optical materials in which light is extremely strongly scattered, the requirement being that the mean free path

![Figure 1.2: Elastic and inelastic scattering rate in unit of $\Gamma$ for different saturation parameter.](image)

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![Figure 1.2: Elastic and inelastic scattering rate in unit of $\Gamma$ for different saturation parameter.](image)
\( \ell_{sc} \) is smaller than the reciprocal of the wavevector, i.e. \( k \ell_{sc} \leq 1 \) (Ioffe-Regel criterion).

In a random laser the multiple scattering process defines optical modes with a certain central frequency and bandwidth, lifetime and rich spatial profile. Random lasers are therefore mirror-less but not mode-less.

1.1.4 The present situation

To develop a theory that describe all aspects of a random laser is very difficult. A complete model would have to include the dynamics of the gain mechanism because gain saturation forms an intrinsic aspect of an amplifying system above threshold [11]. Without gain saturation, the intensity would diverge leading to unphysical results. In addition, as we have seen in the previous section, interference effects have to be included to describe the mode structure.

The need for a detailed model of random lasing became clear after an observation by Cao and co-workers, who found that carefully performed experiments revealed narrow spikes in the emission spectrum on top of a global narrowing [3]. Attempts to understand the origin of these spikes have led to a vivid discussion in the literature. At first it was proposed that Anderson localization could be behind the narrow emission spikes. However, whereas it is possible to obtain localization in lower dimensions [13], it is extremely difficult to obtain it in three-dimensional (3D) systems. The reason is that the Ioffe-Regel criterion \( k \ell_{sc} \leq 1 \) is very difficult to fulfil in optics. Most optical random materials have \( k \ell_{sc} \) values that are much larger than 1.

In a series of recent experiments, it was found that the narrow spikes of a random laser can be observed in nearly the entire range of scattering strengths that is experimentally easily accessible, with \( k \ell_{sc} \) values ranging between 10 and \( 10^4 \) [14]. To better understand the mode structure of a random laser, researchers realized that it was crucial to first understand the modes of passive random materials without gain. To that end, the decay rate statistics of the modes are extremely useful because they provide a clear tool to investigate localization and determine if material sustains localized or extended modes, or both. If gain is then introduced in such materials, it will be the modes with longest lifetime that have the lowest lasing threshold and acquire the highest intensity. Chabanov et al. studied these decay rate statistics experimentally and suggested that long-lived extended modes might be responsible for the observed narrow spikes in random laser spectra [15]. Mujumdar and co-workers then calculated these modes in numerical simulations and found that indeed in a finite-size random system there exists a subset of rare extended modes with very long lifetime, which become very important when gain is introduced. They showed that the “lucky photons” that are spontaneously emitted in such long-lived modes can acquire a huge
Figure 1.3: Emission spectra from a Random laser realized with ZnO semiconductor powder by Cao in 1999 [3]. Excitation intensity grows from bottom to top.
gain and give rise to spikes in the emission spectrum of a random laser [14]. Other experimental works show that extended modes covering the entire sample can become lasing modes and lead to spectrally narrow emission, even if their quality factor is quite low [16]. Although it is currently unknown with certainty whether lasing in zinc oxide, as such studied first by Cao and co-workers, occurs in localized or extended modes, an overall agreement of possible random-lasing mechanisms has come forward. Both extended and localized modes in random systems can lase and lead to spectrally narrow and coherent output. Depending on the experimental geometry, this can be observed as narrow spikes (single shot experiments), or a global narrowing of the spectrum if the narrow spikes are averaged out. Localized modes will also suffer more from gain saturation because they cover a smaller gain volume. The intensity contained in the extended modes on the contrary can grow much higher before the gain is depleted. The debate that has evolved in the literature on this issue shows how rich and complex the physical processes behind multiple scattering in random system are, especially when gain is introduced into the problem.

1.2 Random laser with cold atom at INLN

At the “Institut Non Linéaire de Nice” (INLN, Sophia-Antipolis) a research project is in progress, which aims at realizing a random laser with cold atoms. The project is called CAROL (Cold Atoms for Random Optical Laser). That means Rubidium atoms cooled by lasers and trapped in a magneto-optical trap (MOT) act as active medium and, at the same time, provide multiple scattering.

Laser cooling of neutral atoms has been a very active field of research in the past decades. This was highlighted by the physics Nobel prize in 1997, awarded to S.Chu, C.Cohen-Tannoudji, and W.D.Philips, and by the realization of Bose-Einstein condensation and the corresponding Nobel prize in 2001 to E.Cornell, W. Ketterle, and C. Wieman. This field of research continues to be extremely productive and many interdisciplinary connections are more and more made, such as to quantum computing and solid state physics. Even though quantum gases (Bose-Einstein condensates and degenerate Fermi gases) seem to dominate the scientific production of this field, other interesting effects are studied with cold, albeit non degenerate, atom samples. In the cold-atom group at INLN a new connection between the cold atom community and the field of multiple scattering of light has been initiated. One major goal of this community is to observe and to study strong localization of light (Anderson localization), based on interference effects in multiple scattering. Light (photons) is a very interesting wave to study in comparison to electrons, for which the localization phase transition corresponds to the metal-insulator transition. A clear signature of localization
Figure 1.4: A cloud of Rubidium atoms trapped in the MOT in the centre of the vacuum cell. We can see the fluorescence of the atoms.

of light in a 3 dimensional system remains an open challenge. Cold atoms appear to be promising in this respect, as they have different and advantageous properties compared to standard sample such as white paint (TiO2) or semi-conductor powders. Random laser enters in this research because adding a gain to the passive medium may favour some so-called precursor localized modes even if the strong localization for a passive medium cannot be achieved. In a dense cloud of cold atoms coherent multiple scattering is present and many properties of it have already been studied in the past [5, 4, 17]. We will see in detail in chapter 3 that it is possible to have gain in a Rubidium cold atoms gas. Thus, why not make a random laser with cold atoms? The route along this idea is to combine coherent multiple scattering with gain. As we have seen before, a first step is to realize a random laser with incoherent multiple scattering, the so-called Lethokov photonic bomb, in which recurrent scattering are absent and only a critical volume provides the threshold. The idea is quite simple: when the photon is undergoing multiple scattering events inside the medium it makes a random walk and it is amplified before leaving the medium, a run-away situation will occur, similar to an atomic bomb where there exists a critical mass for the neutrons to be enhanced before leaving the sample. Increasing the path length of the photons by multiple scattering in an active medium can thus lead to a lasing action in a random direction, even though no coherent feedback mechanism,
as in a standard laser cavity, is present.

1.2.1 Advantages of cold atoms

A cloud of cold atoms has very different and advantageous properties compared to the standard medium used to realize random lasers, such as white paint (TiO2) or semiconductor powders. Two main advantages merit to be noticed: the absence of absorption, which in other solid state samples often destroys or at least hides the strong localization of light, and the excellent theoretical knowledge of the microscopic scattering properties of the sample, which often allows for an ab initio quantitative description of the observed signals. Furthermore it is important to mention that the dephasing time $T_2$ of the optical coherence is of the same order as the life time $T_1$ of the excited state population for such cold dilute atomic samples (of the order of several tens of ns); whereas in solid state samples, non-radiative processes (electron-phonon interaction at room temperature e.g.) reduce the life time of optical coherences $T_2$ to orders of magnitude below the population relaxation time: $T_2 \ll T_1$. In addition to these fundamental features pump-probe techniques allow to create a variety of gain mechanisms, which has been already studied in the last years [6, 7] (see also ch. 3), and there are some already developed experimental techniques that allow us to change some important parameter of the sample like its density and its optical thickness (see sec. 2.3).

Of course, cold atoms have also some drawbacks. For example, despite their low temperature, their remaining velocity can induce decoherence (because of the Doppler effect) [18]. Nevertheless, this is not the main challenging problem for the realization of the Letokhov random laser.

1.2.2 Challenging problems

In all the past realizations of random lasers the multiple scattering and the gain could be controlled separately. For example the scattering rate could be increased by adding scattering particles in the active medium, and pumping more or less the medium the gain increases or decreases ($\ell_g$ decreases or increases) without affecting the scattering mean free path $\ell_{sc}$. The CAROL project is challenging in this direction because the cold atoms act, at the same time, as scatterers and as amplifiers. The gain and the multiple scattering are not controlled independently, but are strictly linked. This makes the experimental work harder and opens many questions like: how the scattering rate of the atoms is influenced by the presence of the gain? Or what a photon does when it meets an atom? Is it amplified or is it diffused or both? My work aims to answer these questions.

Furthermore while in the past realizations the random laser emission and the pump laser were well separated in frequency, in our cold atoms experiment it is not easy to distinguish between the pump frequency and
the gain frequency as the typical frequency scale is the megahertz. We can
not filter the pump background light and this makes the detection to be a
big experimental challenge. If we will realize a random laser, how do we
know it? Also on this problematic question I try to answer in my work.

1.2.3 Outline of the manuscript

In the following I will present the realization of the cooling and trapping of
Rubidium atoms both in theory and in our experimental setup (ch. 2). I did
not work on its realization from the beginning but I had to learn how to play
with it and sometime I had to optimize the alignment of laser beams, the
injection of the amplifiers and of the Acousto Optical Modulators (AOM)
or the equilibration of the different beams.

Then I will present some different gain mechanisms allowed in Rubidium
cold atoms gas (ch. 3). I will explain in short the studies already done at
INLN in the past few years on the different gain mechanisms and on the
realization of cavity lasers with them. In particular, I will describe more
deeply the Raman gain, which has been used in my research work.

In the chapters 4 and 5 I will present my work on the threshold predic-
tion and on the tentative of building a complex detection apparatus for the
future random laser signal. In the former I will explain the technique we
implemented to measure the transmission and the diffusion of a weak probe
beam, and the calculation we made in order to extract the significative in-
formation from the measurements, such as the scattering cross section and
the characteristic lengths of the system $\ell_{ac}$ and $\ell_g$. Furthermore I will ex-
plain a simulation we made with Matlab in order to predict the experimental
scattering cross section data starting from the transmission ones. I will give
a value for the threshold of the random laser in term of the on-resonance
optical thickness. In the latter I will talk about the measurement of the
second order correlation function from the power spectrum of the fluores-
cence, I will explain why it is important to build an apparatus that allows
detecting diffused light in a single speckle grain and how we did it, and I will
present the heterodyne detection technique we implemented. I will explain
a calculation of the number of photons expected in the spatial coherence
area and thus which features the detector must have.
Chapter 2

Atom cooling: theory and experimental setup

In the first part of this chapter I will describe the theory of cooling and magneto optical trapping of atoms using laser fields. This technique was proposed at the first time by Wineland and Dehmelt [19] and by Theodor W. Hänsch and Arthur Leonard Schawlow [20] in 1975, and first demonstrated by Letokhov, Minogin and Pavlik in 1976 [21]. Steven Chu, Claude Cohen-Tannoudji and William D. Phillips were awarded the 1997 Nobel Prize in Physics “for development of methods to cool and trap atoms with laser light”. A magneto-optical trap (abbreviated MOT) is a device that cools down non-charged atoms to temperatures near absolute zero and traps them at a certain place using magnetic fields and the optical force of laser light. A MOT requires the atom to have a laser cooling transition in order to work.

In the second part of this chapter I will describe the experimental apparatus I worked on, implemented at INLN in Sophia Antipolis few years ago. The larger part of the apparatus is taken by optical devices to create the laser light with the required power and frequency. Then there are a vacuum chamber containing Rb vapours and coils that provide magnetic field.
2.1 Theory of MOT: laser cooling and magneto-optical trapping

2.1.1 Radiation forces

An atom interacting with electromagnetic radiation exchanges not only energy but also momentum with the field, hence experiences a force affecting its dynamics. This force is used to stop, cool and trap atoms. The momentum exchange between atom and radiation originates, in a quantum framework, from emission and absorption of photons with momentum $\hbar \vec{k}$. In the semi-classical description, the radiation electric field induces on the atom an electric dipole moment. Hence, the Hamiltonian of an atom interacting with a radiation electric field is:

$$H(\vec{r}_e, \vec{p}_e, \vec{R}, \vec{P}, t) = \frac{\vec{P}^2}{2m} + H_0(\vec{r}_e, \vec{p}_e) + \vec{d} \cdot \vec{E}(\vec{R}, t), \quad (2.1)$$

where $(\vec{r}_e, \vec{p}_e)$ are electron variables (we consider hydrogen like atoms, with a single electron in the external shell), $(\vec{R}, \vec{P})$ are center-of-mass variables and $\vec{d} = -e \vec{r}_e$ is the dipole moment of the atom. The internal dynamics of the atoms is described by $H_0$, with eigenstates $H_0 \psi_n = E_n \psi_n$. In the dipole approximation the radiation electric field $E$ is uniform on the atom size (with wavelength $\lambda \gg a_0$, Bohr radius) and depends on the center-of-mass coordinate $\vec{R}$ only.

Since in quantum mechanics $(\vec{R}, \vec{P})$ are quantum operators, with $[R_j, P_k] = i\hbar \delta_{jk}$, their Heisenberg equations are:

$$\frac{d\vec{R}}{dt} = \frac{1}{i\hbar} [\vec{R}, H] = \frac{\vec{P}}{m} \quad (2.2)$$

$$\frac{d\vec{P}}{dt} = \frac{1}{i\hbar} [\vec{P}, H] = -\vec{\nabla}_R H = \vec{\nabla}_R (\vec{d} \cdot \vec{E}) \quad (2.3)$$

where $\vec{\nabla}_R$ is the gradient with respect to the center-of-mass coordinate. Evaluating the average value on the atomic state $\psi$, and defining $\vec{x} = \langle \vec{R} \rangle_\psi$ and $\vec{p} = \langle \vec{P} \rangle_\psi$, the equations become

$$\frac{d\vec{x}}{dt} = \frac{\vec{p}}{m} \quad (2.4)$$

$$\frac{d\vec{p}}{dt} = \langle \vec{\nabla}_R (\vec{d} \cdot \vec{E}) \rangle_\psi. \quad (2.5)$$

If $\vec{E} = \hat{\epsilon} E$ has the polarization $\hat{\epsilon}$, then $d = \vec{d} \cdot \hat{\epsilon}$. Since $E$ does not depend on the internal variable $\vec{r}_e$, it can be factorized out of the quantum average and Eq.(2.5) is approximated by:

$$\frac{d\vec{p}}{dt} \approx \langle d \rangle_\psi \vec{\nabla} E = \vec{F}. \quad (2.6)$$
Hence, the radiation force is proportional to the average atomic dipole and to the electric field gradient. If the field is a monochromatic plane wave, the force is directed along the wave vector $\mathbf{k}$. Moreover, if the radiation intensity varies also, a force directed along the direction of variation occurs. For instance, a focused Gaussian laser beam produces strong transverse and weak longitudinal forces which can be trapping or expulsing, depending on the the atom response, i.e. on the average atomic dipole

$$
\langle d \rangle_\psi = d_{12}(\rho_{12} + \rho_{21}),
$$

where $d_{12}$ is the dipole element of the transition (we consider only two-level atoms) and $\rho_{12}$ is the coherence of the atomic density matrix.

**Scattering force**

For a plane wave, $E = E_0 \cos(kx - \omega t)$, the dipole force is:

$$
F_x = -kd_{12}E_0(\rho_{12} + \rho_{21}) \sin(kx - \omega t)
$$

$$
= -(\hbar k)\Omega(\rho_{12} + \rho_{21}) \sin(kx - \omega t),
$$

where $\Omega = d_{12}E_0/\hbar$ is the Rabi frequency. The density matrix elements evolve following the equations:

$$
\dot{\rho}_{12} = -i(\omega - \omega_0 - kv_x)\rho_{12} - (\Gamma/2)\rho_{12} + (i/2)\Omega(\rho_{22} - \rho_{11})
$$

$$
\dot{\rho}_{22} = -\dot{\rho}_{11} = -\Gamma \rho_{22} + (i/2)\Omega(\rho_{12} - \rho_{21})
$$

where

$$
\rho_{12} = \rho_{12}e^{i(kx-\omega t)}
$$

(we consider a closed transition, with $\rho_{11} + \rho_{22} = 1$, with only radiative decay, i.e. $\gamma_2 = \Gamma$ and $\gamma_{12} = \Gamma/2$). When the relaxation time $\Gamma^{-1}$ is much shorter than the characteristic time of variation of the atomic momentum (typically $\mu$s against $\mu$s), then

$$
\rho_{12} \approx \frac{i\Omega}{\Gamma} \frac{\rho_{11} - \rho_{22}}{1 + i(2\Delta/\Gamma)}
$$

$$
\rho_{22} \approx \frac{(\Omega/\Gamma)^2}{1 + (2\Delta/\Gamma)^2 + 2(\Omega/\Gamma)^2},
$$

and

$$
\rho_{11} - \rho_{22} \approx \frac{1 + (2\Delta/\Gamma)^2}{1 + (2\Delta/\Gamma)^2 + 2(\Omega/\Gamma)^2}.
$$

where $\Delta = \omega - \omega_0 - kv_x$. From (2.11) and (2.12) we obtain

$$
\rho_{12} + \rho_{21} = \frac{-2\Omega/\Gamma}{1 + (2\Delta/\Gamma)^2 + 2(\Omega/\Gamma)^2} \left\{ \frac{2\Delta}{\Gamma} \cos(kx - \omega t) + \sin(kx - \omega t) \right\}
$$

(2.15)
Inserting (2.15) in the expression (2.8) of the force and averaging in time, taking into account that
\[
\sin(kx - \omega t) \cos(kx - \omega t) = 0 \quad \text{and} \quad \sin^2(kx - \omega t) = 1/2,
\]
we obtain
\[
F_x = \frac{\hbar k}{\Gamma} \frac{\Omega^2}{1 + (2\Delta/\Gamma)^2 + 2(\Omega/\Gamma)^2}.
\] (2.16)

Finally, defining the saturation intensity \( I_s \) such that:
\[
\frac{2\Omega^2}{\Gamma^2} = \frac{I}{I_s},
\] (2.17)
then the scattering force (2.16) assumes the form:
\[
F_{\text{scatt}} = \hbar k \frac{\Gamma}{2} \left( \frac{I}{I_s} \right)^2 + \frac{I}{I_s}.
\] (2.18)

The scattering force equals the rate at which the absorbed photons impart momentum to the atom, i.e. \( F_{\text{scatt}} = (\hbar k) \Gamma_{sc} \), where
\[
\Gamma_{sc} = \Gamma \rho_{22} = \frac{\Gamma}{2} \frac{I}{I_s} \left( \frac{1}{1 + 4(\Delta/\Gamma)^2 + I/I_s} \right).
\] (2.19)

is the scattering rate, which is maximum at resonance, \( \Delta = 0 \). The maximum force is for \( I >> I_s \), \( F_{\text{max}} = \hbar k(\Gamma/2) \), when \( \rho_{22} \approx (1/2) \). It has the following interpretation in terms of photons: each absorbed photon gives the atom a kick in the direction of the laser beam propagation and spontaneously-emitted photons go in all directions, so that this contribution is averaged to zero. The scattering force is able to deflect or stop an atomic beams, producing an acceleration \( a_{\text{max}} = v_{\text{rec}}/\tau \), where \( v_{\text{rec}} = \hbar k/m \) is the recoil velocity and \( \tau = \Gamma^{-1} \) is the lifetime of the excited state. Typically for alkali atoms \( v_{\text{rec}} \) is few mm/s (6mm/s for Rb) and \( \tau \) tens of ns, so \( a_{\text{max}} \sim 10^5 \) times the gravitational acceleration \( g \). The typical stopping distance of an alkali atomic beam with a velocity \( v_0 \sim 1000 \text{ m/s} \) is \( L \sim 1 \text{ m} \).

However, the atoms experiencing such a force are only those in a narrow range of velocities, \( \Delta v \sim \Gamma/k \), for atoms having a range of Doppler shift approximately equal to the homogeneous width of the transition, \( \Gamma \). Atoms that interact strongly with the laser light slow down until the change of their Doppler shift takes them out of the resonance with the light. It is possible to compensate this change in order to keep the force close to its maximum throughout the slowing process (see the Zeeman slower and the chirp cooling techniques [22]).
Dipole force

We have seen that the mechanism of scattering force relies on the absorption of photons incident on the atom. The analogous classical force is the radiation pressure force \( F = IA/c \), when light of intensity \( I \) is absorbed by a surface \( A \). In fact, the momentum carried by the light is \( E/c \) where \( E \) is the light energy and, since the power is \( P = dE/dt \), the light transfers momentum to the surface at rate \( d(E/c)/dt = IA/c \). However, classically it exists another kind of radiation force not related to absorption, arising from the deflection of the light by a dispersive medium: for instance, when light passes through a prism of refraction index \( n \), it is deviated by an angle proportional to \( n \). The angular deviation can be seen as a change of the light momentum in direction and not in modulus, and induces a reaction force on the prism itself. More specifically, if the change of the light direction is \( \theta \), the change of the wave vector is \( \Delta k = 2k\sin(\theta/2) \) and the force caused by the light on the prism is \( (IA/c)2\sin(\theta/2) \). This force is known as dipole (or gradient) force, and has interesting applications, for instance in biology (‘optical tweezers’) [22]. When a small dielectric sphere is illuminated by a focused laser beam, it diffracts light on opposite sides of the sphere with different strengths, proportional to the spatial varying intensity of the laser. This causes a net force that pull the sphere toward the region of high intensity, i.e. near the focus. With this method, it is possible to drag micro-organisms in water, as for instance biological cells, without perturbing them.

In order to derive the expression of the dipole force on a two-level atom, we return to the radiation force (2.6), for a plane wave in which the electric field amplitude varies also on \( x \). Then, Eq.(2.8) becomes more generally:

\[
F_x = (\rho_{12} + \rho_{21}) \left[ -(h\Delta)\Omega \sin(\omega t - kx) + h\Omega \frac{\partial \Omega}{\partial x} \cos(\omega t - kx) \right] = F_{\text{scatt}} + F_{\text{dip}}.
\] (2.20)

Combined with the atomic dipole (2.15) and after the average in time, the dipole force is:

\[
F_{\text{dip}} = \frac{1}{2} \left( \frac{\partial \Omega^2}{\partial x} \right) \frac{h\Delta}{\Gamma^2 + (2\Delta/\Gamma)^2 + 4(\Omega/\Gamma)^2} + \frac{h\Delta}{2I_s \partial x} \frac{1}{1 + (2\Delta/\Gamma)^2 + I/I_s}.
\] (2.21)

The dipole force is proportional to the intensity gradient and is zero on resonance. For \( \Delta \gg \Gamma \) (and an intensity such that \( \Delta \gg \Omega \)), the dipole force is:

\[
F_{\text{dip}} \approx -\frac{\partial}{\partial x} \left( \frac{h\Omega^2}{4\Delta} \right).
\] (2.22)
Hence, the force derives from a potential and more generally, in three dimensions

\[ \vec{F}_{\text{dip}} = -\vec{\nabla}U_{\text{dip}}, \]

where

\[ U_{\text{dip}} \approx \hbar \Omega^2 \frac{I}{4\Delta} = \hbar\Gamma \frac{I}{8\Delta I_s}. \]

### 2.1.2 Laser Cooling and Trapping: MOT

Whereas an atomic beam can be slowed with a single laser beam, a gas needs three orthogonal pairs of lasers to reduce its temperature (cooling). It takes advantage of the \textit{Doppler effect} to imbalance the forces of two counter-propagating laser beams with the same frequency.

Let’s consider initially a laser beam on an atom moving with a velocity \( v \) along the lasers direction. The detuning is \( \Delta = \Delta_0 \mp kv \) where \( \Delta_0 = \omega - \omega_0 \) and the minus (plus) sign holds for atoms moving in the same (opposite) direction of the laser beam. The force, for \( I \ll I_s \) and low velocities, \( kv \ll \Gamma \), is:

\[ F_{\text{scatt}} = \hbar k \frac{\Gamma}{2} \frac{I/I_s}{1 + (2\Delta_0/\Gamma)^2} \approx F_0 \mp \alpha v \]

where

\[ F_0 = \hbar k \frac{\Gamma}{2} \frac{I/I_s}{1 + (2\Delta_0/\Gamma)^2} \]

and

\[ \alpha = 4 \hbar k^2 \frac{I}{\Gamma I_s} \frac{-\Delta_0}{(1 + (2\Delta_0/\Gamma)^2)^2} \]

The second term acts as a viscous force (proportional to the velocity) if \( \Delta_0 < 0 \) (red-shifted detuning). If on the atoms act two counter-propagating laser beams with the same frequency and intensity, the net force is

\[ F_{\text{molasses}} = (F_0 - \alpha v) - (F_0 + \alpha v) = -2\alpha v \]

which is a friction force if \( \alpha \propto -\Delta_0 > 0 \). With three orthogonal pairs of lasers red-detuned from the atomic resonance it is possible to cool an atomic gas to very small temperature (hundreds or tens of \( \mu K \)). In the reference frame of the atom moving toward the right, the Doppler effect leads to an increase in the frequency of the laser beam propagating in the direction opposite to the atom’s velocity. This Doppler-shift brings the light closer to resonance with the atom and thereby increasing the rate of absorption from this beam. This leads to a resultant force that slows the atom down. The light induces a frictional, or damping, force on the atoms just like that on a particle in a viscous fluid (like honey). For this analogy, this way to cool atoms is called \textit{Optical Molasses technique}. 

23
The optical molasses technique accumulate cold atoms in the region where the three orthogonal laser beams intersect, because it takes a long time for atoms to diffuse out. With some change in the polarization of the laser beams and adding a magnetic field gradient, this configuration can be turned into a trap (Magneto-Optical Trap). The magnetic field is created by two pair of coils around the atoms, with current in opposite directions, producing a quadrupole magnetic field, which is zero at the center of the coils and whose magnitude increases linearly in every direction for small displacements from the zero point. The magnetic field does not confine atoms by itself, but it produces a variable Zeeman shift of the atomic hyperfine levels. The Zeeman shift plays the same role of the Doppler effect of the previous paragraph because it causes an unbalance in the scattering forces of the counter-propagating laser beams depending, this time, on the atomic position and not on the atomic velocity. It is thus the radiation force which strongly confines the atoms. For a $J = 0 \rightarrow J = 1$ transition, the constant magnetic field gradient splits the three sub-level with $M_J = 0, \pm 1$ of $J = 1$ with a separation depending on the atom’s position. The counter-propagating laser beams have circular polarization and red-shifted frequency with respect to the atomic resonance (see figure 2.1). The frequency shift

![Figure 2.1: Principle of the trapping force in a MOT. The magnetic field remove the degeneracy of the $J = 1$ level. The energy of the sublevels $m_J$ depends linearly on the position because of the quadrupole magnetic field. The two counter-propagating laser beam are polarized $\sigma^+$ and $\sigma^-$, thus the transition is allowed only toward one of the sub-levels. The laser beams frequency is red-shifted respect to the atomic resonance so the resultant force is toward $r = 0$. Thanks to the Doppler effect a friction force appear also because the atom absorbs more photons from the beam propagating towards it (effect not represented in the figure).](image)

caused by the magnetic field can be incorporated in the detuning of the
scattering force, $\Delta = \omega \mp kv - (\omega_0 \pm \beta z)$, so that for two laser beams with opposite circular polarizations,

$$F_{\text{MOT}} = F^\sigma_{\text{scatt}}(\Delta_0 - kv - \beta z) - F^\sigma_{\text{scatt}}(\Delta_0 + kv + \beta z) \approx -2 \frac{\partial F}{\partial \Delta_0}(kv + \beta z) = -\alpha v - \frac{\alpha \beta}{k} z,$$

(2.28)

where $\beta = (g \mu_B / \hbar)(dB/dz)$.

The unbalance of the radiation force caused by the Zeeman effect leads to a restoring force with a spring constant ($\alpha \beta / k$). Atoms that enter the region of intersection of the laser beams are slowed and the position-dependent force pushes the cold atoms to the trap center, providing an efficient method to load a large number of cold atoms (up to $10^{10}$), to be used in experiments.

**Optical Dipole Trap**

If we look at the expression (2.24) for the dipole potential we can see that when $\Delta$ is positive ($\omega > \omega_0$) this potential has a maximum where the intensity is highest, i.e. the atoms are repelled from regions of high intensity. But in the opposite case, of $\Delta$ negative ($\omega < \omega_0$), the dipole force acts in the direction of increasing $I$ and $U_{\text{dip}}$ is an attractive potential: atoms in a tightly-focused laser beam are attracted towards the region of highest intensity, both in the radial direction and along the axis of the beam. This dipole force confines the atoms at the focus of a laser beam to create an optical dipole trap. Let’s note that in this case the force is conservative, i.e. there is not a dissipation mechanism, like in the optical molasses technique, where the exceeding atomic momentum is carried away by the spontaneously-emitted photons. We note that the dipole potential is proportional to $I/\Delta$ whereas the radiation pressure is proportional to $I/\Delta^2$: increasing the detuning of the laser the dipole force became more and more important respect to the radiation pressure.

It is thus possible to trap atoms also thanks to the dipole force illuminating, for example, a cloud of cold atoms with a gaussian beam.


2.2 Experimental setup

The experimental setup I worked on at the “Institut Non Linéaire de Nice” (INLN, Sophia-Antipolis) has already been described in detail in [23] and [24]. I will remind the most important features.

![Image of experimental setup]

Figure 2.2: The photo shows the experimental setup in the INLN cold atoms laboratory. It is possible to see the complexity of the optical apparatus necessary to obtain the high power and the frequency control requested for the laser beams. See the simplified scheme of the laser set-up in the figure 2.3. For explanation see [24] and [23].

2.2.1 The Laser System

The complex laser set-up shown in the figure 2.3 and in the figure 2.2 is necessary to obtain flexible time and frequency control of all lasers and to obtain the relatively high power requested to stop and trap a large number of atoms. Fundamentally in our experiment we trap $^{85}$Rb atoms using the $F = 3 \rightarrow F' = 4$ close transition of the $D_2$ line ($5S_{1/2} \rightarrow 5P_{3/2}$) (see figure 2.4). We lock the MOT master DBR diode laser (Yokogawa YL78XN) on the cross-over (see saturated absorption technique in [25]) between the lines $F = 3 \rightarrow F' = 4$ and $F = 3 \rightarrow F = 2$. Thus it is locked at about 90 MHz below the interesting transition and it is afterwards blue shifted via a double pass in an Acousto Optical Modulator ($AOM1$ in the figure 2.3). Finally
the MOT lasers have a frequency red shifted of about 3Γ from the transition. Then we inject a slave diode (SDL5401-G1) which provides us with more than 20mW, and subsequently a tapered amplifier (SDL-8630-E). The laser passes through a second acousto optical modulator (AOM 2) that acts as a switch (ON/OFF) for the MOT laser beam. On the first order we have about 300mW. This is the laser beam that after the splitting in six equal part reaches the Rubidium cell to make the molasses. In order to increment the capture volume in the cell each MOT beam is enlarged up to a waist of \( w = 2.4 \text{ cm} \) using a telescope.

As we perform the atomic cooling by using the \( F = 3 \rightarrow F' = 4 \) slightly red-detuned by \( \sim 3\Gamma \) we also have a non null probability to excite the \( F = 3 \rightarrow F' = 3 \), which is an open transition. That means that some atoms can reach the other ground state \( F = 2 \). In order to re-pump these atom in the \( F = 3 \) hyperfine ground state we need another laser tuned to excite the \( F = 2 \rightarrow F' = 3 \) transition. Otherwise the atoms in the hyperfine ground state \( F = 2 \) are lost.

In our setup we use also for the “repumper” a master-slave configuration with a DBR master (Yokogawa YL78XNW/S) and a SDL slave (SDL-5401-G1). The master laser is locked to a cross-over transition between \( F' = 2 \) and \( F' = 3 \) and shifted by a double pass AOM (AOM4 in the figure 2.3) before injecting the slave laser. The output beam of the slave laser passes
Figure 2.4: Energy levels of $^{85}\text{Rb}$ and $^{87}\text{Rb}$. The transition used to trap the atoms is the $F = 3 \rightarrow F' = 4$ of the D2 transition of $^{85}\text{Rb}$. The transition used for the repumper is the $F = 2 \rightarrow F' = 2$ or 3 of the same D2 transition line.

through an AOM (AOM5) in single pass configuration allowing to switch the repumper laser on and off.

We control the time sequence of the MOT switching on and off the lasers by sending a TTL signal to the AOM2. During the off phase the AOM2 is off, thus all the laser light is in its zero diffraction order. This laser light could be used for pump-probe experiments. The zeroth order of the AOM2 passes indeed through the AOM3 (see the figure 2.3) that acts as a switch, and its $-1$ order is used as a pump laser beam. Its frequency is controlled by the AOM1. A part of the pump beam injects another slave diode and hence become the probe beam. It can be detuned from the pump laser thanks to another double pass in the AOM6.

2.2.2 The MOT

The six MOT beams reach the Rubidium cell in counter propagating pairs in the $\sigma^+ - \sigma^-$ configuration (see figure 2.1). The MOT is loaded from a vapor of Rubidium $^{85}\text{Rb}$ atoms at room temperature in a cubic quartz cell of size 10cm. A system of vacuum valves connects the cell to an ionic pump (25l/s), which allows a vacuum of $10^{-8} \text{mbar}$ inside the cell, and to a reservoir of Rb. A good work pressure is about $5 \cdot 10^{-6}\text{mbar}$. A magnetic field gradient of 10G/cm is obtained by two coils in anti-Helmholtz configuration. We are also able to switch off such a magnetic field in about 50$\mu$s.
2.2.3 The temporal sequence

We are able to control the temporal sequence of the trap. Typically we use a cyclic time configuration in which we have the trap lasers and magnetic field switched on for 29ms and off for 1ms. The off phase is not long enough to lose the trapped atoms, thus in this phase it is possible to do some pump-probe experiments on the unperturbed cold atoms cloud. The cyclic configuration allows a very fast averaging procedure.

2.2.4 Typical MOT features

With the experimental setup described above, we are able to cool the atoms down to a temperature of few tens of µK and we can trap up to $10^{10}$ Rubidium atoms (density $\eta = 10^{10} \text{at/cm}^3$). The typical size of the cloud $L$ is 1cm and its shape is almost spherical. We can obtain optical thickness (see sec. 2.3 for explanation) between 10 and 100 varying the charging time or the detuning of the MOT lasers.
2.3 Measurement of the optical thickness of a cloud of cold atoms

We have seen previously that in order to build a random laser it is crucial to have an optically thick sample. It is possible to measure the optical thickness of a sample from a transmission measurement because

\[ T = \frac{I_{\text{out}}}{I_{\text{in}}} = e^{-\frac{L}{\ell_{sc}}} = e^{-b(\delta)}, \]  

(2.29)

where \( I_{\text{in}} \) and \( I_{\text{out}} \) are the intensity of the probe laser respectively at the input and output, \( b(\delta) \) is the optical thickness, \( \ell_{sc} \) is the scattering mean free path for the passive medium and \( L \) is the length of the sample. It is clear that if the optical thickness is much larger than 1, then \( \ell_{sc} \ll L \) and so we are in the diffusive regime.

In order to derive the equation 2.29 we take the steady state equation of the field in the Maxwell-Bloch equations:

\[ \frac{\partial \Omega}{\partial z} = -\frac{\omega_p^2 T_2}{c} \frac{\Omega}{1 + \delta^2 T_2^2 + T_1 T_2 \Omega^2} \]  

(2.30)

where \( \omega_p = \sqrt{\frac{\eta d^2 \omega_0}{2\hbar c \omega_0}} \) is the plasma frequency, \( \delta = \omega - \omega_0 \) is the detuning and \( \Omega = \frac{4\pi}{k} \) is the Raby frequency. Neglecting the collision, for cold atoms we have \( T_1 = T_2 = \frac{1}{2} \Gamma \), and introducing the saturation intensity

\[ \frac{I}{I_{\text{sat}}} = T_1 T_2 \Omega^2 = \frac{2\Omega^2}{\Gamma^2}, \]  

(2.31)

the equation 2.30 becomes

\[ \frac{\partial I}{\partial z} = -\alpha \frac{I}{1 + 4 \frac{\delta^2}{\Gamma^2} + \frac{I}{I_{\text{sat}}}}, \]  

(2.32)

where \( \alpha = \frac{4\omega_p^2 c^2}{\Gamma^2} \) is the diffusion coefficient. Using

\[ d = \sqrt{\frac{3\pi \varepsilon_0 \hbar \Gamma}{k^3}}, \]  

(2.33)

the well known cross section at resonance

\[ \sigma_0 = \frac{3\lambda^2}{2\pi} \]  

(2.34)

and the definition of \( \omega_p \), the diffusion coefficient become

\[ \alpha = \eta \sigma_0, \]  

(2.35)
where $\eta$ is the atomic density. For $I \ll I_{\text{sat}}$ the equation 2.32 yields

$$I(L) = I(0)e^{-b(\delta)},$$

(2.36)

where

$$b(\delta) = \frac{b_0}{1 + 4\delta^2}$$

(2.37)

and $b_0 = \eta\sigma_0 L$. We have found the result 2.29. We can introduce a characteristic length $\ell_{sc}(\delta)$ that is the scattering mean free path and, at resonance, is $\ell_{sc} = \frac{1}{\eta\sigma_0}$.

![Figure 2.5: Optical thickness. The measurement corresponds to $b_0 = 20$.](image)

Experimentally we measure the optical thickness at resonance $b_0$ of a cloud of cold atoms by sending a weak tunable probe laser on the sample and measuring the intensity in transmission, on the other side of the cloud, with a photodetector. We change the detuning of the probe laser around the atomic resonance in order to obtain a graph of the transmission in function of the detuning of the probe. We obtain the value of $b_0$ from the full width of the curve at half maximum. In fact, calling the width at half maximum $\Delta\nu_{FWHM}$, from equation 2.29 and imposing $T = 1/2$ we obtain the optical thickness at resonance

$$b_0 = \ln(2)(1 + \frac{\Delta\nu_{FWHM}^2}{\Gamma^2}).$$

(2.38)

In the figure 2.5 we can see a typical measure of the optical thickness.
In principle the equation 2.29 with \( b(\delta) \) as defined in 2.37 is valid only for a monochromatic laser source. Of course, in real experiments this is not the case. We can calculate this correction taking into account the spectral width of our laser. If we suppose a Lorentzian spectral shape \( \mathcal{L}(\delta \nu) \) we can write the transmission as

\[
T(\delta) = \int_{-\infty}^{+\infty} \mathcal{L}(\delta \nu) e^{-b(\delta + \delta \nu)} d\delta \nu.
\]  

(2.39)

By the numerical integration of the equation 2.39 we calculate \( \Delta \nu_{FWHM} \) for different optical thickness in the non monochromatic case. The figure

![Figure 2.6](image)

Figure 2.6: **Correction of the optical thickness for the non monochromatic case.**

2.6 for a laser width of 2 MHz shows that the measurement of the optical thickness could be overestimated if we do not take into account the laser linewidth.

In our experimental setup, after a good alignment of the laser beams, we reach optical thickness of some tens. And applying known compression techniques we can reach even \( b_0 \sim 100 \). This means that, at least in the passive medium, we can reach a strongly diffusive regime where the photons make a very large number of scattering events (\( N_{\text{scatt-events}} \sim b^2 \)) before escaping from the surface of the cloud. The next step is to introduce the gain in a so strong diffusive medium (see chapter 3).
Chapter 3

Gain with cold atoms

The atomic vapors are well known medium with a strong non-linearity in the atoms-radiation interaction. The study of hot vapors by pump-probe spectroscopy is limited by the Doppler effect. In the past Mollow gain effect has been studied in strongly collimated atomic jets exploring the direction transverse to the motion. In this one dimensional case the Doppler effect did not play. Raman effect was a well known gain effect just for non-atomic medium.

The development of magneto-optical traps has opened the possibility of cooling atomic vapors, thus reducing the Doppler effect and so allowing to work with laser frequencies close to atomic resonance. This lead to the experimental observation of Raman and Mollow gain effects in the transmission spectra of a probe laser through cold atoms clouds.

At the “Institut Non Linéaire de Nice” (INLN) in Sophia Antipolis (France) it has been studied in the past few years these gain mechanisms and also an other gain mechanisms based on the non-linear effect of four wave mixing (FWM) [6], and it has been realized cavity lasers with all of these kinds of gain [7]. We talk about gain because a probe is amplified taking energy from a pump laser. It is important to note that all these processes create amplification without a population inversion in the bare-atom basis. It is possible to explain these amplification effects by multi-photons processus [26]. I will present (in detail) the Raman gain process, the one I used in my work.

3.1 Raman Gain: a two-photons process

The Raman effect is a processus that involves two levels with a small energy difference compared to the energy difference with a third one, e.g. two hyperfine levels or two Zeeman levels.

We consider the diffusion process in a medium with a ground state with a double degeneracy. Then, the inelastically scattered light contains fre-
frequencies called Stokes and anti-Stokes at \( \omega \pm \omega_R \), where \( \hbar \omega_R \) is the energy difference between the two groundstate sublevels \(|a\rangle\) and \(|b\rangle\). The figure 3.1 shows these two processes.

![Figure 3.1: Raman scattering process. (a) Stokes. (b) Anti-Stokes.](image)

In a pump-probe experiment it is possible to induce also a stimulated Raman diffusion process. It is a two-photon transition, e.g. from \( J = 1 \) to \( J = 2 \) ground states, that removes pump photons and creates new probe photons without exciting the atom. This kind of transition does not require a population inversion between the ground and the excited states. The transition between two different \( J \) sub-levels requires orthogonal polarization between the pump and the probe. The first observation of this phenomena was made quite casually in 1962 by Woodbury [27], and later explained by Eckhardt et al. [28]. In the particular case in which the pump laser is tuned close to an atomic transition between a ground state and an excited state, both with multiple degenerate Zeeman sub-levels, it is possible to observe the stimulated Raman effect involving the light-shift (which removes the degeneracy of the energy levels) and the optical pumping process (which leads to a population inversion), both caused by the intense pump laser.

For Rubidium atoms we consider the transition \( F = 3 \rightarrow F' = 4 \) of the \( D2 \) transition (see 2.4). With a pump laser we can excite the transitions of the Zeeman structure of the \( F = 3 \) level. The condition is that the pump-probe detuning \( \delta \) is equal to the energy difference between the two Zeeman sub-level coupled by a two-photons transition. The amount of level degeneracy can be due to a residual magnetic field or to the light-shift induced by the pump. A full explanation of the light-shift and of the so-called dressed states can be found in [12]. In the case of a residual magnetic field the Zeeman energy shift is

\[
E_B = g \mu_B m_F |B|, \tag{3.1}
\]

where \( g \) is the Landé factor \((g = -1)\), \( \mu_B \) the Bohr magneton \((\mu_B = \)
$-1.4\, MHz/Gauss$ and $m_F$ is the magnetic quantum number ($m_F = 0, \pm 1, \pm 2, ...$). In the case of large pump detuning $\Delta \gg \Gamma$ the light-shift is

$$E_L \propto \frac{\Omega_p^2}{4\Delta},$$

where the proportionality factor is linked to the Clebsch-Gordan coefficients, that are different for different sublevels (see Appendix A). If we consider a pump detuning $\Delta = 4.5\Gamma$ the light-shift is weak and the Raman resonances are displaced by approximately $500 kHz$, and so the Raman spectrum is much narrower with respect to the Mollow one ($\sim \Gamma = 6 MHz$). In the figure 3.2 we can see an intuitive image of the process (we call the sub-levels $|F, m_F\rangle$ also for light shifted levels). A full explanation of the two Raman

![Figure 3.2: Scheme of the transition $F = 3 \rightarrow F' = 4$ of the D2 transition of $^{85}Rb$. The light shift or the magnetic field break the degeneracy. Many Raman two-photons transitions are possible when the pump-probe detuning correspond to the energy difference between two subsequent Zeeman sub-levels. According to the population of each Zeeman sub-level, it could be absorption or gain on the probe. In the case of the displacement due to the light shift the energy difference $\delta$ changes for each transition because of different Clebsch-Gordan coefficients.](image)

diffusion process (spontaneous and stimulated) can be found in [29]. The Raman transmission spectra of a weak probe can be expressed as $T = e^{g(\delta)L}$ where the linear gain is

$$g(\delta) \propto \frac{\Omega_p^2}{\Delta^2} \left( \frac{a\gamma}{(\delta + \omega_R)^2 + \gamma^2} - \frac{b\gamma}{(\delta - \omega_R)^2 + \gamma^2} \right),$$

(3.3)
where $\omega_R$ and $\gamma$ are the frequency difference and the width of the resonances, while $\Delta$ and $\Omega_P$ are the pump detuning from the atomic transition and the pump Rabi frequency, and $\delta$ is the pump-probe detuning. The Raman spectrum is thus composed by two resonances, both with a Lorentzian shape with different weights ($a$ and $b$) and signs. One represent the absorption and the other the gain.

If the pump detuning is red-shifted $\Delta < 0$ then we have gain for the pump-probe detuning $\delta < 0$, while if the pump detuning is blue-shifted $\Delta > 0$ the gain is when $\delta > 0$. This is due to different sub-level populations in the two cases (see figure 3.3). In first approximation, the width of the Raman resonances is $\Omega_P^2 \Delta^2$ and the displacement between them is on the order of $\Omega_P^2 \Delta$ [29].

In figure 3.4 I show the study made by Franck Michaud on the displacement of the Raman resonances by adding a magnetic field in different directions [26]. A detailed experimental study of Raman gain in Rubidium cold atoms can be found in [6] and in [26].

### 3.2 Mollow gain and Four Wave Mixing gain

There are other possible gain mechanisms obtainable by pump-probe techniques on cold atoms. Mollow gain has been studied theoretically for the first time by Mollow in 1972 [30]. It can be explained using the Optical Bloch equations for a two level atom driven by two laser fields: a pump and a probe. It is possible, in certain condition, to obtain an amplification of the probe beam [30, 26, 31]. This gain process involves a real transition to the upper excited state (in our case the transition $F = 3 \rightarrow F' = 4$ of the $D2$ transition of $^{85}$Rb) so the resonance is of the order of the decay rate of the...
transition (Γ). Mollow gain is possible to obtain for a small pump detuning and with a pump and a probe beams with the same linear polarization. I show in figure 3.5 an example of Mollow spectrum for \( \Delta = \Gamma \).

The Four Wave Mixing (FWM) is a kind of parametric gain. It is a third order non linear process and it consists in an interaction between two counterpropagating pump beams (F and B) and a probe beam (p). The result is the formation of a fourth beam, called conjugated (c), that satisfies both the frequency matching and the phase matching, which are \( \omega_c + \omega_p = \omega_F + \omega_B \) and \( \vec{k}_c + \vec{k}_F + \vec{k}_B + \vec{k}_p = 0 \). In the configuration with two counterpropagating pump beams \( \vec{k}_F = -\vec{k}_B \), thus \( \vec{k}_c = -\vec{k}_p \) so the conjugated beam propagates in the opposite direction respect to the probe. A classical image of the phenomena consists in the formation of an optical grating thanks to two of the beams, and the third beam is reflected or diffracted and creates the conjugated one. It exists a configuration in which the conjugated beam is higher than the probe beam and so there is a gain in reflection. A study made by F. Michaud and G. Gattobigio [6] varying the probe detuning gives the spectra of the conjugated beam. The result is shown in figure:

In conclusion the cloud of cold atoms is a good gain medium. During the last year W. Guerin and F. Michaud realized cavity lasers with all of this kind of gains [7]. In the figure 3.7 I report the different transverse cavity modes they obtained.
Figure 3.5: An example of Mollow spectra for $\Delta = \Gamma$. Note that the spectra is broadened respect to the Raman one.

Figure 3.6: An example of Non Degenerate Four Wave Mixing spectra in the configuration $F//B \perp p$ for different probe detunings $\delta$ [6].
Figure 3.7: Transverse modes of the different cold-atoms lasers [7]. (a) Gaussian profile obtained by inserting a diaphragm near the waist of the cavity. Typical modes of: (b) the Mollow laser, (c) the Raman laser, and (c) the four-wave mixing laser.

3.3 Gain and scattering

We have seen in this chapter that it is possible to obtain gain in our cold atoms gas exploiting different gain mechanisms.

In the section 2.3 we have seen that we are able to obtain with the cold atoms a sample with very strong scattering properties. In the passive medium (without pumping) we are able to reach a multiple scattering regime. In the previous section we have seen that it is possible to add a gain in the cold atoms sample by pumping them with strong laser fields.

The next step to obtain a random laser is to combine gain and multiple scattering. My work has been carried out in this context. As we have already said in the section 1.2.2 this is challenging because for cold atoms random laser the atoms act, at the same time, as scatterers and as amplifiers. None has already studied this problem experimentally. The adding of pump lasers in the system changes the scattering properties of the atoms. My work aimed to understand this modification, especially for the Raman gain. In the next chapter I will report the experimental measurement of the scattering mean free path $\ell_{sc}$ and of the gain length $\ell_g$ in the pumped sample, and the subsequent calculation of the random laser threshold.

As the gain frequency is close to the pump frequency (less than few $MHz$) the detection of the scattering from a weak probe and, in future, the detection of the random laser emission are an experimental challenge. The atomic fluorescence is the good signal to look at although it is usually hard to see it.

In the measurement of the scattering from a weak probe the problem is how to distinguish the few photons scattered from the probe in the background of fluorescence from the pump. We marked the probe photons by
modulating the probe beam in intensity. In the power spectrum of the total atomic fluorescence the peak at the modulation frequency correspond to the probe scattered photons. In the next chapter I will explain this technique more in detail.
Chapter 4

Scattering rate with gain: threshold prediction

In a cavity laser the prediction of lasing threshold is done sending a weak probe laser and measuring separately the losses of the empty cavity and the double pass gain of the active medium varying the pumping power. The threshold is reached when gain become equal to losses. Likewise for a random laser we need to measure the single pass gain and the total losses from the surface of the cloud of atoms, i.e. the scattering in all directions of a weak probe laser. Measuring the single pass gain is simple because it is a transmission measurement. It is harder to measure the losses. These measures let us to extract the two characteristic lengths of the sample (the scattering mean free path $\ell_{sc}$ and the gain length $\ell_g$) and thus to predict the random laser threshold using the equation B.1.

The equivalent for the random laser of the standard empty cavity is the cloud of atoms as a passive medium; because, as we have seen before, for a random laser it is the scattering that provide the losses and the feedback. We cannot measure the losses from the passive medium because the presence of the pump changes the atomic response (for gain and also for scattering). Therefore we need to measure the amount of scattered probe photons in presence of the pump.

4.1 How to measure the scattered photons?

While the measure of gain needs only the input and output intensity of the probe, the measure of the scattering is harder because the light scattered from the weak probe in the direction of the detector is very weak in comparison to the light scattered from the pump. The problem is that we need to have at the same time gain in the medium and scattering. Increasing the pump the gain increases too, but also the number of the pump photons increases. In principle, on the other hand, we need a weak probe because
the amount of gain change also with the input intensity (Figure: 4.1). We

Figure 4.1: Raman gain for different probe saturation parameter $s_p$. Measured by Gattobigio G. and Michaud F. see [6]. $s_p$ is defined in 1.7.

choose the best trade-off between these two limitations: pump power in order to have gain and probe intensity $I \approx I_{sat}$. Also in this configuration the light scattered from the probe is not enough to distinguish it from the pump light. So we need to mark the probe photons in a way we can distinguish them in the detection. The idea we have implemented is to modulate in intensity the input probe with a known low frequency, so in the power spectrum of the detected signal we have a peak at the modulation frequency as we can see in the figure 4.2. In other words, we make a kind of synchronous detection. We know that this modulation creates two lateral bands in the laser spectrum at frequencies $\omega_L + \omega_M$ and $\omega_L - \omega_M$ from the simple relation:

\[
E_0 \frac{\alpha}{2} \cos(\omega_M t) \cos(\omega_L t + \phi_L) = E_0 \frac{\alpha}{4} \cos((\omega_L - \omega_M) t + \phi_L) + E_0 \frac{\alpha}{4} \cos((\omega_L + \omega_M) t + \phi_L)
\]

(4.1)

where $\omega_L$ and $\omega_M$ are the laser and modulation frequencies, and $\alpha$ is the modulation peak-peak amplitude. In the power spectrum we have a peak at the modulation frequency that depends on $\alpha$, so we need an higher modulation to have an higher peak.

It is important that the modulation frequency is much lower than the width of the Raman gain, otherwise the probe photons are not sensible to the Raman structure. Since the Raman width is 300–500 kHz we chose $\omega_M = 50 kHz$, and we modulated at least at 60% ($\text{visibility} = 0.6$). We
have verified that the Raman gain was not affected by this modulation.

We plot, as \( S \) for “scattering”, the height \( h \) of the modulation peak measured in presence of the pump (see figure 4.2). To take into account only the scattered probe, we subtract the background noise from the pump (measured without probe or by averaging the points besides the modulation peak). Moreover, we normalize by the peak height without pump. Finally,

\[
S = \frac{\sqrt{(h_{\text{pump+probe}}(V^2) - h_{\text{pump}}(V^2))}}{h_{\text{probe}}(V)},
\]

(4.2)

where \( h \) is the height of the peak at the modulation frequency in \( V \) or in \( V^2 \) as specified; note that in the figure 4.2 the power spectrum is in decibel (dB).

At the same time we measured the transmission of the probe. Measuring the intensity in input and in output of the cloud of cold atoms and calculating the transmission as follows

\[
T = \frac{I_{\text{out}}}{I_{\text{in}}},
\]

(4.3)
4.2 The measurements

4.2.1 Transmission and scattering rate vs pump power: atomic saturation

We make these two measurements, first of all, for different values of pump power and for two different probe detunings: the first one in correspondence of the maximum gain and the other in a point without gain (see the circle in the figure 4.3). We report the transmission and the scattering rate respectively in the figures 4.4 and 4.5.

We can see in figure 4.4 that the gain grows with the pump power, and that the transmission without gain is higher for higher pump power, due to atom saturation. From the figure 4.5 it is clear that the scattering rate decreases with the pump power but the role played by the Raman gain is not clear. The decaying of the scattering rate is well explained by the atomic saturation as if it was a two-level atom. From the well known formula for the scattering rate in a two level atom,

\[ \Gamma_{sc} = \frac{\Gamma}{2} \frac{I/I_{sat}}{1 + 4 \frac{\Delta^2}{\Gamma^2} + \frac{I_{pump}}{I_{sat}}} \]  \hspace{1cm} (4.4)

we derive the scattering rate for a weak probe in presence of an intense pump by considering a little variation \( d\Gamma_{sc} \) of equation 4.4. That is:

\[ d\Gamma_{sc} = \frac{\Gamma}{2} \frac{dI}{I_{sat}} \frac{1 + 4 \frac{\Delta^2}{\Gamma^2} + \frac{I_{pump}}{I_{sat}}}{(1 + 4 \frac{\Delta^2}{\Gamma^2} + \frac{I_{pump}}{I_{sat}})^2} \]  \hspace{1cm} (4.5)

Figure 4.3: Raman gain and Transmission with and without pump. The circle are the positions of the measurement.
Figure 4.4: Transmission for different pump power. In black: transmission for $\delta$ probe in correspondence of gain. In red: transmission for $\delta$ probe on the right of the Raman structure (without gain).

Figure 4.5: Scattering Rate for different pump power normalized with the modulation peak without pump. In black: scattering for $\delta$ probe in correspondence of gain. In red: scattering for $\delta$ probe on the right of the Raman structure but without gain.
And so the ratio between the scattering rate of the probe with and without pump is:

$$\frac{d\Gamma_{\text{with pump}}}{d\Gamma_{\text{without pump}}} = \left(1 + 4\frac{\Delta^2}{\Gamma^2}ight)^2 \left(1 + 4\frac{\Delta^2}{\Gamma^2} + \frac{I_{\text{pump}}}{I_{\text{sat}}}ight)^2$$

(4.6)

We plot the corresponding curve on the graph of the figure 4.5.

### 4.2.2 Transmission and scattering rate vs probe detuning around the Raman structure

In order to understand better what happens to the scattering rate when we add a gain, we fix the power of the pump and we study transmission and scattering rate, as described above, changing probe detuning across the Raman structure and not only for two values. In other words we make a spectrum. We obtain a curve like that on figures 4.6, 4.7, 4.8.

![Figure 4.6: Probe Transmission and Scattering for different probe detuning around the Raman structure. Parameters: $\Delta = 4, 5\Gamma$; $I_{\text{pump}}/I_{\text{sat}} = 10$; $I_{\text{probe}}/I_{\text{sat}} = 1, 1$; $b_0 = 12$; Modulation frequency = 50kHz. Magnetic field switched off.](image)

The difference between these three curves is the value of the magnetic field used to change the spacing between the amplification and absorption lines.

We can see in these plots that the scattering rate is strongly modified by the Raman structure. For detuning values beside the Raman structure there is a reduction of the scattering rate due to the pump intensity as predicted.
Figure 4.7: Probe Transmission and Scattering for different probe detuning around the Raman structure. Parameters: $\Delta = 4, 5\Gamma$; $I_{\text{pump}}/I_{\text{sat}} = 9, 4$; $I_{\text{probe}}/I_{\text{sat}} = 1, 1$; $b_0 = 17$; Modulation frequency = 40kHz. A weak magnetic field is switched on in order to separate the absorption and the amplification structures. For magnetic field configuration see figure 3.4 (b).

Figure 4.8: Probe Transmission and Scattering for different probe detuning around the Raman structure. Parameters: $\Delta = 4, 5\Gamma$; $I_{\text{pump}}/I_{\text{sat}} = 14$; $I_{\text{probe}}/I_{\text{sat}} = 1, 2$; $b_0 = 19$; Modulation frequency = 60kHz. A strong magnetic field is switched on in order to separate the absorption and the amplification structures. For magnetic field configuration see figure 3.4 (b).
by the eq. 4.6. In order to understand better the influence of the Raman structure on the scattering rate, we apply a magnetic field parallel to the probe electric field using the configuration of the figure 3.4 (b). We can clearly see the separation of the scattering rate into two peaks. We note also that these scattering peaks are shifted respect to the transmission ones: it seem that the maximum and the minimum of the transmission correspond to the maximum slope of the scattering rate.

The points $S_{corr}$ correspond to a correction of the measured points $S$, they represent the scattering cross section as we derive in the following.

### 4.3 Extracting information from Transmission and Scattering measurement: $\ell_{sc}$ and $\ell_g$

We have measured at the same time the gain and the losses from the atomic sample. Now, we have to extract from these measurements the two characteristic lengths: $\ell_{sc}$ and $\ell_g$.

We know from the optical theorem that in a passive medium the cross section of transmission $\sigma_{tr}$ is equal to the cross section of scattering $\sigma_{sc}$:

$$\sigma_{tr} = \sigma_{sc}. \quad (4.7)$$

This can be explained with a simple physical image: in absence of absorption or amplification (passive medium) the photons lost in transmission are scattered, because in a passive medium the total number of photons is conserved. When we add gain or absorption (active medium), the photons lost in transmission could be scattered or absorbed by the medium and vice-versa, if we consider a negative absorption cross section we have photons added in transmission and so we have gain. The equation 4.7 for an active medium becomes:

$$\sigma_{tr} = \sigma_{sc} + \sigma_{abs} = \sigma_{sc} - \sigma_g \quad (4.8)$$

where $\sigma_{abs}$ is the absorption cross section and $\sigma_g$ is the positive gain cross section. From the relation:

$$\sigma = \frac{1}{\eta \ell} \quad (4.9)$$

with $\eta$ the sample density, and $\ell$ the characteristic length of the considered process, equation 4.8 becomes:

$$\frac{1}{\ell_{tr}} = \frac{1}{\ell_{sc}} - \frac{1}{\ell_g}. \quad (4.10)$$

where $\ell_{tr}$ is called transmission length. In this treatment we are neglecting the probability that a scattered photon return on the straight original path after other two diffusions.
Figure 4.9: Link between measured fluorescence and $\ell_{sc}$ in a single scattering approximation. In our setup $\theta \simeq 19^\circ$.

The simple transmission measurement gives us

$$T = \frac{I_{\text{out}}}{I_{\text{in}}} = e^{-\frac{L}{\ell_{tr}}} = e^{-\frac{L}{\ell_{tr}} + \frac{1}{\ell_g}}. \quad (4.11)$$

It is clear that a single transmission measurement is not enough to extract $\ell_{sc}$ and $\ell_g$. We need also the measure of the scattering rate. But what have we really measured in diffusion?

We consider a single scattering regime, we call $x$ the position on the straight path at which the scattering event happens, and $d(x, \theta)$ the path length the photon travels in the direction of the detector before escaping from the atomic cloud (see the figure 4.9). From simple geometrical calculation

$$d(x, \theta) = \left(\frac{L}{2} - x\right) \cos(\theta) + \sqrt{\left(\frac{L}{2} - x\right)^2 \cos^2(\theta) - (x^2 - Lx)} \quad (4.12)$$

where $\theta$ is the angle at which the photon is scattered. So the scattering signal is proportional to

$$S_{\text{nonorm}} \propto \frac{1}{\ell_{sc}} e^{-\frac{x+d}{\ell_{tr}}}. \quad (4.13)$$

where the first term takes into account the scattering cross-section, and the exponential term is the transmission on a path of length $x+d$. We normalize with the diffusion of the probe in absence of the pump. In this case $1/\ell_g = 0$
and \( \ell_{sc} = \ell_{sc0} \). The measured signal is therefore

\[
S = \frac{\langle \frac{1}{\ell_{sc}} e^{-\frac{x+d}{L}} \rangle}{\langle \frac{1}{\ell_{sc0}} e^{-\frac{x+d}{L_{sc0}}} \rangle}
\]  

(4.14)

where \( \langle \cdots \rangle = \frac{1}{L} \int_0^L \cdots dx \) is the average on the position of the scattering event, with \( L \) the diameter of the cloud. Using the definition of transmission 4.11, we obtain

\[
S = \frac{\ell_{sc0}}{\ell_{sc}} \times \frac{\langle TP^{-\frac{x+d}{L}} \rangle}{\langle T_{noP}^{-\frac{x+d}{L}} \rangle}.
\]

We are interested in the first term, that is the modification of the scattering mean-free path due to the pumping. We define \( S = S_{corr} \times C \), with

\[
S_{corr} = \frac{\ell_{sc0}}{\ell_{sc}},
\]

(4.15)

and

\[
C = \frac{\langle TP^{-\frac{x+d}{L}} \rangle}{\langle T_{noP}^{-\frac{x+d}{L}} \rangle}
\]

(4.16)

is a correction factor.

We make a Matcad numerical evaluation to calculate this correction factor. We plot in figure 4.10 \( \langle T^{-\frac{x+d}{L}} \rangle / T \) vs \( T \). We can see in figure 4.10

\[\text{Figure 4.10: Matcad numerical simulation: Correction factor. We plot } \langle T^{-\frac{x+d}{L}} \rangle / T \text{ vs } T. \text{ In red the simulation without considering light polarization. In blue the simulation taking into account the randomization of the polarization after the scattering.}\]

50
that the red curve is very close to 1 in all range of transmission values we have. And so the correction factor is well approximated by

\[ C \simeq \frac{T_P}{T_{noP}}. \]  

(4.17)

This is because in our configuration the angle \( \theta \) is very small and so \( x+d \simeq L \).

Till now we have neglected that after diffusion the polarization of the photon is randomized and so the Raman gain, that is maximum for orthogonal polarization (respect to the pump), is less efficient. We can consider this effect by multiplying for \( \langle \cos^2(\phi) \rangle = 1/2 \) the gain in the path of length \( d \) in the equation 4.13. This become

\[ S_{nonorm} \propto \frac{1}{\ell_{sc}} e^{-\frac{\ell_{tr}}{\ell_{sc}} - \frac{\ell_{tr}^2}{\ell_{g}} + \frac{\langle \cos^2(\phi) \rangle d}{\ell_{g}}}. \]  

(4.18)

The correction factor is more complicated than before and the approximation 4.17 is not correct any more (see the curve blue in figure 4.10). We can calculate it only numerically. Dividing the measured scattering signal \( S \) by the correction factor we obtain

\[ S_{corr} = \frac{\ell_{sc0}}{\ell_{sc}} \]  

(4.19)

where \( \ell_{sc0} \) is known from the measure of the optical thickness \( \frac{L}{\ell_{sc0}} = b(\delta) \) (see section 2.3). In the figures 4.6, 4.7, 4.8 of the section 4.2.2 we have plotted \( S_{corr} \) in green.

From the equations 4.11 and 4.19 we can calculate the two characteristic lengths \( \ell_{sc} \) and \( \ell_{g} \):

\[ \frac{\ell_{sc}}{L} = \frac{1}{b(\delta)S_{corr}}; \]  

(4.20)

\[ \frac{\ell_{g}}{L} = \frac{1}{\ln(T) + b(\delta)S_{corr}}. \]  

(4.21)

And so we can write the Letokhov threshold B.1 as

\[ b(\delta)S_{corr} (\ln T + b(\delta)S_{corr}) > \frac{4\pi^2}{3} \]  

(4.22)

We remember that \( S_{corr} \) is the scattering rate normalized and corrected by the propagation effects in the cloud, but this correction is correct only in a single scattering regime, and that \( b(\delta) \) is the optical thickness at the frequency we work (see eq. 2.37). Another way of writing the threshold is to use a critical on-resonance optical thickness instead of a critical cloud size \( L \). We make the strong hypothesis that the measured atomic response (scattering and gain) does not depend on \( b_0 \) but only on the pump parameters, i.e. \( \ln(T) \propto b_0 \) and \( S_{corr} \) does not depend on \( b_0 \). Thus we can write

\[ b_{0\text{cr}} = 2\pi \left[ 3S_{corr0} \left( \frac{\ln T}{b_{0\text{meas}}} - S_{corr0} \right) \right]^{-1/2}, \]  

(4.23)
where $S_{corr0} = \frac{S_{corr}}{1+\frac{1}{2} (\Delta+\delta)^2}$, with $\Delta$ the pump detuning from atomic resonance and $\delta$ the pump-probe detuning.

Probably this hypothesis is not verified for large optical thickness, because of the light diffused by the atoms in the neighborhood, as Davidson et al. proved in 1999 for Mollow gain [32]. The atoms are illuminated not
only by the direct light of the pump laser beam, but also by elastically and 
inelastically scattered light from the other atoms having different directions, 
frequencies and polarization. There is a sort of gain saturation that we can 
write as 
\[ \ln T = \frac{A b_0}{1 + B b_0} \] 
with \( A \) and \( B \) two real parameter depending on pump intensity \( \Omega \) and de-
tuning \( \Delta \). In the limit of higher and higher optical thickness \( \ln T \sim A/B \).

If we solve the equation 4.22 in this limit, and keeping the hypothesis that 
\( S_{\text{corr}} \) does not depend on \( b_0 \), we find 
\[ b_{0,cr} = \frac{1}{2S_{\text{corr}}} \left( \sqrt{\left(\frac{A}{B}\right)^2 + \frac{16\pi^2}{3} - \frac{A}{B}} \right). \]

From a first measure of Raman gain for different optical thickness we found 
\( A = 0.02905 \) and \( B = 0.01703 \) by fitting the data with the formula 4.24 for 
the pump intensity and the detuning we use in our measurement.

With our experimental measurement we obtain \( b_{0,cr} \sim 250 - 500 \) but 
we are strongly limited in exploring the range of parameters \( \Delta \) and \( \Omega_{\text{pump}} \), 
and we are using a probe that is not so weak. If we increase the pump 
power, or if we reduce the probe intensity, the noise of the pump exceeds 
the modulation peak of the probe in the power spectrum, thus the scattering 
rate is not measurable any more.
4.4 Kramers-Kronig relations

Looking at the transmission and scattering spectra we note that they look like absorption-dispersion curves. They are in fact not independent because the Kramers-Kronig relations make a link between the real and imaginary parts of \( \alpha \). The transmission is related to the imaginary part of the atomic polarizability \( \alpha(\omega) \) and the scattering cross section is proportional to \(|\alpha(\omega)|^2\).

For a complex function \( \chi(\omega) = \chi_1(\omega) + i\chi_2(\omega) \) of the complex variable \( \omega \), analytic in the upper half plane of \( \omega \) and which vanishes as \(|\omega| \to \infty\), \( \chi_1(\omega) \) and \( \chi_2(\omega) \) are linked by the Kramers-Kronig relations:

\[
\chi_1(\omega) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\chi_2(\omega')}{\omega' - \omega} d\omega',
\]

(4.26)

and

\[
\chi_2(\omega) = -\frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\chi_1(\omega')}{\omega' - \omega} d\omega',
\]

(4.27)

where \( P \) is the Cauchy principal value. We see that the real and imaginary parts of such a function are not independent, so that the full function can be reconstructed given just one of its parts. We know that the required condition of analiticity is fulfilled by the causality, i.e. the susceptibility as a temporal function must take into account only the past and not the future.

Now, the medium susceptibility is \( \chi = \eta \alpha \), where \( \eta \) is the atomic density of the sample and \( \alpha \) is the atomic polarizability. So we can extract \( Im[\alpha(\omega)] \) from the transmission measurement \( (T = e^{-k_0 Im(\alpha)L}) \) and then we can calculate \( Re[\alpha(\omega)] \) through K-K relation, and finally calculate \(|\alpha(\omega)|^2\). More deeply the scattering cross section is

\[
\sigma_{sc} = \frac{k_0^4}{6\pi}|\alpha|^2,
\]

(4.28)

and the transmission cross section is

\[
\sigma_{tr} = k_0 Im(\alpha),
\]

(4.29)

where \( k_0 \) is the wave number in the vacuum. And so the gain cross section is

\[
\sigma_g = -k_0[Im(\alpha) - \frac{k_0^3}{6\pi}|\alpha|^2].
\]

(4.30)

We made a simulation with our data. We wrote a matlab program that fit the transmission data and through K-K relations calculate the expected scattering. The fit on Raman gain is done adding two Lorentzian on the imaginary part of the atomic polarizability of a two level atom written by
Mollow [30], in the form rewritten by Luis Froufe-Pérez [31].

$$\alpha_{Mollow}(\omega) = -\frac{\varepsilon}{2} \frac{|z|^2}{|z|^2 + \frac{\Omega^2}{2}} \left( \Gamma_0 + i\delta \right)(z + i\delta) + \frac{i\Omega_p^2\delta}{2z} \left( \Gamma_0 + i\delta \right)(z^* + i\delta) + |\Omega_p|^2(\Gamma_0/2 + i\delta)^2,$$

where

$$z = \Gamma_0/2 - i\Delta$$

$$\Delta = \omega_0 - \omega_p$$

$$\delta = \omega_p - \omega_s$$

with \( \omega_0 \) the atomic resonance, \( \omega_p \) and \( \omega_s \) the pump and the probe frequencies. We report the result in the figures 4.14, 4.15 and 4.16.

Figure 4.14: Matlab simulation on the data of figure 4.6. In blue the fit of the transmission data and in red the calculation through K-K of the scattering cross section.

We note that we are quite able to predict the scattering rate from measurement of the transmission.

### 4.5 Limits and comments

We have implemented an experimental technique that allows us to measure the two characteristic lengths of the active medium \( \ell_{ac} \) and \( \ell_g \). We know from Letokhov how to calculate the threshold from these lengths. We remember that Letokhov formula is valid in the diffusive regime. But we note that,
Figure 4.15: Matlab simulation on the data of figure 4.7. In blue the fit of the transmission data and in red the calculation through K-K of the scattering cross section.

Figure 4.16: Matlab simulation on the data of figure 4.8. In blue the fit of the transmission data and in red the calculation through K-K of the scattering cross section.
even if the optical thickness at resonance is high (from 10 to 20 in our measurements), at the detuning we work and with the saturation induced by the pump it results that $\ell_{sc} > L$ as we can see for example in figure 4.13. It is clear that the corresponding prediction will not be accurate. On this, a work is in progress to obtain a threshold prediction for random lasers, which would be valid with much less stringent parameters. This new theoretical approach is based on the use of the radiative transfer equation instead of the diffusion equation, following the ref [33].

Another important limit is that we can do this kind of measurement only in a limited range of the parameters ($\Omega_{pump}, \Delta, \Omega_{probe}$) because of the low frequency noise of the pump, that in the diffusion measurement can exceed the height of the probe modulation peak. It is thus important to improve our ability in fitting the transmission data and in calculating through K-K relations the scattering rate.

We note that in the simulation the scattering rate prediction is lower than the data in the middle of the Raman structure. This could be explained. In the Raman structure we have both an absorption and an amplification peak. The first one add some inelastic diffusion because of the absorption re-emission process. And in principle the second one not because the pump and probe saturation parameter is lower than unity (about elastic and inelastic scattering, see the section 1.1.3). From Optical Bloch Equations we know that the elastic scattering is related to the coherences of the density matrix $\rho_{12}$, while the inelastic scattering is proportional to the population of the excited state $\rho_{22}$ [34]. We know that $\alpha(\omega)$ comes from $\rho_{12}$ (see [30]). So with the simulation we calculate only the elastic part of the diffused light. With this kind of measurement we can not distinguish between elastic and inelastic scattering and with the simulation we predict only the elastic part. But for a random laser it is better to have elastic scattering that preserves the frequency (as a mirror does). The inelastic part can lower the threshold because it adds diffused photons, but this is not interesting for our purpose.

Now, with the analytical calculation of diffusion we have a procedure that allow us to explore a larger range of the parameters ($\Omega_{pump}, \Delta, \Omega_{probe}$). This allow us to optimize the gain using higher pump intensity and lower probe intensity, and to reduce the detuning of the pump $\Delta$, in order to try to reach the diffusive regime where the Letokhov formula works better.

### 4.6 Conclusion

We are able to measure simultaneously the gain and the diffusion for a limited range of parameters. We can measure the transmission and predict through an analytical calculation, the diffusion in the entire range of the pump-probe parameters. The measured quantities allow us to extract the
requested quantities to calculate the threshold.

The pump saturation effect reduces the scattering but the Raman structure increases the scattering instead of reducing it. The latter is a good news looking at the Random laser project because we need to have the more diffusion as possible. But we are still far from the random laser threshold and also from the amplified spontaneous emission: the diffusion with active medium is always lower than without pump.

Finally we can say that the predicted threshold is not impossible to reach with the state-of-the-art experimental techniques. The next steps are the threshold prediction for the other gain mechanisms and subsequently the realization of the random laser, so the reaching of this threshold.

In the following chapter I will remember the detection problems and I argue some possible technique to detect the random laser emission.
Chapter 5

The detection of the future random laser emission

The detection of the cold atoms random laser emission is currently a challenge. The closeness of the emission frequency and the pump frequency creates big experimental problems, because of the impossibility of filtering the pump fluorescence light. Another problem is the weakness of the random laser signal respect to the intense pump scattered light (at least near the threshold).

The fluorescence is the laser light diffused in all the directions by the trapped atoms. It is known from the literature [35] that it is possible to extract, from the atomic fluorescence, the velocity distribution of the atoms in the trap, that is related to their temperature. We can extract similar information from the photon statistics, from the temporal correlation functions of the light and from the light beating spectroscopy (known also as Diffusing Wave Spectroscopy) [36]. We are interested in exploiting these diagnostic techniques for the random laser detection.

The interesting random laser signature could be found in the optical power spectrum or in the noise power spectrum of the atomic fluorescence. The optical power spectrum can be measured in different ways. A large optical spectrum (\(\sim \Gamma\)), like the Mollow one, can be measured using a Fabry-Perot. But for the detection of a narrower optical spectrum, like the Raman one, we must implement other techniques like the heterodyne or the homodyne ones. The noise power spectrum can be measured with a self-beating detection technique or, in an other framework, measuring the intensity correlation function \(g^{(2)}(\tau)\) rather than the photon statistics directly by counting the photons. I will discuss the link between the noise power spectrum and the second order correlation function.

I will not talk about the photon statistics but I will explain the intensity correlation function and the heterodyne detection technique. The intensity correlation function may change its features below and above threshold, and
in the optical spectrum, it may appear, above the pump noise background, a peak due to the random laser emission.

Since we are implementing beating detection techniques we can not neglect the spatial coherence of the fluorescence light. I will explain the importance of measuring the fluorescence light in a single speckle grain.

5.1 Homodyne and heterodyne detection technique

An idea to detect the random laser emission is to measure the optical spectrum of the diffused light. When the system exceeds the threshold, the emission occurs at the gain frequencies. For the cold atoms case we have already seen that the random laser emission frequency will be close to the pump frequency. An idea to distinguish between them is to look at the optical spectrum. Above threshold new frequencies must appear close to the pump laser frequency. A way to obtain the optical spectrum of the light is the so called heterodyne detection technique.

Figure 5.1: Scheme of the principle of the heterodyne detection technique. The light diffused by the cold atoms at the frequency of the input laser beam $\omega_L$ and the random laser emission at frequency $\omega_{RL}$ beat with a local oscillator of frequency $\omega_L + \delta \omega_{LO}$. The local oscillator is derived from the probe beam and detuned using an Acousto Optical Modulator (AOM) while the incident beam is the pump laser. The probe derives from the pump (see figure 2.3).

The heterodyne detection is a well known beating technique. It consists in the beating onto the detector between the light field of the source at frequency $\omega$ and the field of a more intense local oscillator at frequency $\omega_{LO}$. The beating at frequency $\omega_{LO} - \omega$ contains information about the scattered optical field. This technique has been largely used in the past.
in the study of fluorescence light from atomic molasses and atomic lattices [35, 37].

I show in figure 5.1 the scheme of the principie of the heterodyne technique. The local oscillator is derived from the probe beam at frequency $\omega_L$. The incident beam on the atoms is the pump. Since the probe is derived from a part of the pump beam (see the figure of the experimental setup 2.3), they have a well defined relative phase. Without this condition the beating is averaged to zero.

The fundamental property of the heterodyne technique is that the obtained noise intensity spectrum is an exact copy of the optical spectrum of the scattered light, but shifted by the local oscillator frequency value $\omega_{LO} = \omega_L + \delta\omega_{LO}$:

$$P_{\text{hetero}}(\omega) \propto i_{LO}P_0(\omega - \omega_{LO}) + \text{terms}, \quad (5.1)$$

where $i_{LO}$ is the constant photocurrent due to the local oscillator. $P_0$ is the shifted optical spectrum centered in the adjustable frequency $\delta\omega_{LO}$. The other terms in the sum are the white noise and the continuous component (at zero frequency). In the case in which the local oscillator has the same frequency of the incident laser this technique is called homodyne.

Another beating technique is the self beating spectroscopy that allows to relate the spectral properties of the photocurrent to the spectral properties of the light field [38] with the convolution

$$P_i(\omega) \propto \int_0^{\infty} P_0(\omega')P_0(\omega + \omega')d\omega' \quad (5.2)$$

where $P_i(\omega)$ is the power spectrum of the photocurrent $i$, and $P_0(\omega)$ is the optical power spectrum. The self-beating occurs for example onto the detector where the different components of the optical field beat between them. The condition of validity of eq. 5.2 is that the light must have a gaussian statistic, so when the Siegert relation works (see eq. 5.8). On the contrary the equation 5.1 for the heterodyne technique works always for coherent sources.

The self-beating detection gives the same information than the measurement of the intensity correlation function or the photon statistics [36]. In the following section I will explain in detail the intensity correlations and how they are related to the optical power spectrum.

5.2 Measurement of the second order correlation function

We consider the simple model of a medium constituted by $N$ ($N >> 1$) identic and independent atoms lightened by a laser beam of frequency $\omega_L$.
Figure 5.2: Simple explanation of atomic fluorescence. A laser beam incident from direction $\vec{k}_{in}$ is diffused by the atoms in the detection direction $\vec{k}_{obs}$.

incident with a direction $\vec{k}_{in}$ (see figure 5.2). We consider laser light with a weak saturation parameter $s << 1$, so that we can neglect the inelastic diffusion (see par. 1.1.3). We treat the light as a classical electromagnetic field, and we suppose that the distance between the scattering medium and the detector is much larger than the sample size. The field observed in the direction $\vec{k}_{obs}$ in the position $\vec{r}_{det}$ is the sum of $N$ contributions of the form

$$E_j(t) = A_j e^{-i\omega L t} e^{i \vec{q} \cdot \vec{r}_j} e^{i \vec{k}_{obs} \cdot \vec{r}_{det}}, \quad (5.3)$$

where $1 \leq j \leq N$ and the phase $\phi_j = (\vec{k}_{in} - \vec{k}_{obs}) \cdot \vec{r}_j = \vec{q} \cdot \vec{r}_j$ depend on the scatterer position and on the observation geometry. The amplitude $A_j$ is a real positive quantity independent on the scatterer, so $A_j = A$, and $\vec{r}_{det}$ is the detector position. The total field irradiated is

$$E(t) = \sum_{j=1}^{N} E_j(t) = Ae^{-i\omega L t} e^{i \vec{k}_{obs} \cdot \vec{r}_{det}} \sum_{j=1}^{N} e^{i \vec{q} \cdot \vec{r}_j(t)}. \quad (5.4)$$

We suppose that the detector is not placed in the direction of the incident beam, so $\vec{q} \neq 0$. The field correlation function (not normalized) is

$$C_E(\tau) = \langle E(t)E^*(t+\tau) \rangle = A^2 \left\langle \sum_{j,k=1}^{N} e^{i \vec{q} \cdot (\vec{r}_j(t)-\vec{r}_k(t+\tau))} \right\rangle, \quad (5.5)$$

where the average $\langle \cdot \cdot \cdot \rangle$ is done on the random variables $\vec{r}_j$ and $\vec{r}_k$. If the $N$ atoms are independent and randomly distributed, the phases $\vec{q} \cdot \vec{r}_j(t)$ are equally distributed in the interval $[-\pi, \pi]$. The sum on $j$ and $k$ is a
sum of complex random terms non correlated. Each term of the sum is zero except for the case \( j = k \). Hence, for \( t = 0 \) and zero delay \((\tau = 0)\), the correlation function for zero delay is \( C_E(0) = NA^2 \) and it is proportional to the average light intensity \( \langle I(t) \rangle \). The normalized field correlation function become

\[
g^{(1)}(\tau) = \frac{C_E(\tau)}{\langle I(t) \rangle} = \alpha \exp \left( - \vec{r} \cdot (\vec{r}(t+\tau) - \vec{r}(t)) \right),
\]

(5.6)

with \( \alpha \) a real number. The average \( \langle \cdots \rangle \) is made on the ensemble of possible trajectory \( \vec{r}(t) \) of a single generic atom. It is important to note that the field correlation function reflects the temporal evolution of the external degree of freedom of the atoms through the function \( \exp \left( - \vec{r} \cdot (\vec{r}(t+\tau) - \vec{r}(t)) \right) \) [36]. This function is called “self scattering function” and often appears in the studies of particle dynamics or of fluids. It reflects the temporal fluctuations of the phase of the diffused field by an atom.

We can also calculate the second order correlation function (intensity correlation function) defined as

\[
g^{(2)}(\tau) = \frac{\langle E(t)E^*(t+\tau)E(t+\tau)E^*(t) \rangle}{\langle E(t)E^*(t) \rangle \langle E(t+\tau)E^*(t+\tau) \rangle}.
\]

(5.7)

With similar calculations it is possible to find the well known Siegert relation

\[
g^{(2)}(\tau) = 1 + |g^{(1)}(\tau)|^2,
\]

(5.8)

valid for a light field resultants from the sum of many random and mutually independent contributions (light with a gaussian statistic).

The intensity correlation function is related to the power spectrum of the diffused light through the Wiener-Khinchin theorem

\[
C_I(t) = \int_{-\infty}^{\infty} |I_\omega|^2 e^{i\omega t} d\omega = 3^{-1} \left[ |I_\omega|^2 \right],
\]

(5.9)

where \( C_I \) is the non normalized intensity correlation function, and \( |I_\omega|^2 \) is the light power spectrum. The Doppler effect enlarges the laser light spectrum and the random laser emission adds new frequencies to the spectrum. From the knowledge of the intensity correlation function it is possible to obtain the optical power spectrum of the fluorescence light. For example, it allows to measure the Doppler broadening associated with the atomic velocity distribution. Thus it is possible to calculate the temperature \( T \) of the trapped atoms [35] because of

\[
\langle v^2 \rangle = \frac{k_B T}{m},
\]

(5.10)

where \( k_B \) is the Boltzmann constant and \( m \) is the atomic mass.
5.3 Importance of temporal and spatial coherence

The Siegert relation is valid just for an ideal detector, that means a detector without any spatial extension and with an infinitely large bandwidth. In real experiences these two conditions are not fulfilled. Also in the measurement of the optical power spectrum, using the heterodyne detection technique explained above, we have supposed to work with a spatially coherent light. Since the fluorescence from the atoms forms a speckle pattern, the light does not have spatial coherence.

I will introduce the concepts of spatial and temporal coherence in order to estimate precisely the spectrum shape and correlation function really measured.

The intensity correlations take into account the degree of temporal coherence through the correlation function $g^{(2)}(\tau)$, but also its spatial coherence properties via a space-temporal function taking into account the spatial extension of the detector and of the diffusive medium itself. The spatial effects must be well understood in order to choose the best geometrical configuration for the detection. A rigorous explanation of these concepts in interferometry can be found in the chapter 5 of [39] and in [40]. Here I summarize the most important concepts.

Let consider an extended monochromatic source with a radiative surface $A_{\text{source}}$. The degree of complex coherence of the light $\gamma(\vec{r}_1, \vec{r}_2, \tau)$ is a measure of spatial and temporal coherence of the light irradiated by the source in two observation points $M_1$ at position $\vec{r}_1 \equiv (x_1, y_1)$ and $M_2$ at position $\vec{r}_2 \equiv (x_2, y_2)$. We can write

$$\gamma(\vec{r}_1, \vec{r}_2, \tau) = \frac{\langle E^-(\vec{r}_1, t + \tau) E^+(\vec{r}_2, t) \rangle}{\sqrt{\langle I(\vec{r}_1) \rangle \langle I(\vec{r}_2) \rangle}}; \quad (5.11)$$

the quantity

$$\mu(\vec{r}_1, \vec{r}_2) = \gamma(\vec{r}_1, \vec{r}_2, 0) \quad (5.12)$$

is the spatial coherence factor and it characterizes the spatial effects only. We can calculate it through the Van Citter-Zernike theorem. It says that $\mu(\vec{r}_1, \vec{r}_2)$ is proportional to the spatial Fourier transform of the source luminance distribution. In other words, each point of a light source produces, in a double-slit Young experiment, an interference figure where the contrast is $\mu(\vec{r}_1, \vec{r}_2) = 1$. In the case of an extended source and under the hypothesis of spatial incoherence of the source itself, the light irradiated by different source points interfere because of different spatial phases and the resultant interference figure is more noisy. The contrast of the fringes $\mu(\vec{r}_1, \vec{r}_2)$ is thus a sum of exponentials with different phases.

In the general case it is not possible to separate the spatial and temporal effects in the coherence function $\gamma(\vec{r}_1, \vec{r}_2, \tau)$, and the optical spectrum evolves during light propagation [39]. Nevertheless, in most real situations,
\( \gamma(\vec{r}_1, \vec{r}_2, \tau) \) can be factorized in the product of two functions representing the spatial and the temporal coherence. In these conditions the optical spectrum is invariant during propagation [41]. A sufficient condition is the spatial incoherence of the source and that the optical spectrum emitted by each point is the same. In other terms, we need an homogeneous medium without spatial correlations. We can suppose, in first approximation, to be in these conditions.

Finally, we integrate the spatial coherence effects in the calculation of the temporal correlation function for an extended detector. We call \( g^{(2)}(\vec{r}_1, \vec{r}_2, \tau) \) the temporal correlation function of the intensity detected in two points of the detector: \( M_1 \) at time \( t \) and \( M_2 \) at time \( t + \tau \). We have thus the generalized Siegert relation [41]

\[
\begin{align*}
  g^{(2)}(\vec{r}_1, \vec{r}_2, \tau) &= 1 + |\gamma(\vec{r}_1, \vec{r}_2, \tau)|^2 \\
  &= 1 + |\mu(\vec{r}_1, \vec{r}_2)|^2 |g^{(1)}(\tau)|^2. \\
\end{align*}
\]

Integrating on the detector surface we obtain a factor called temporal contrast, that is

\[
\eta(A_{\text{det}}, A_{\text{source}}) = \int_{\text{detector}} |\mu(\vec{r}_1, \vec{r}_2)|^2 d^2 \vec{r}_1 d^2 \vec{r}_2 / A_{\text{det}}^2. \tag{5.14}
\]

Thus the temporal correlation function for an extended detector is

\[
g^{(2)}(\tau) = 1 + \eta(A_{\text{det}}, A_{\text{source}}) |g^{(1)}(\tau)|^2. \tag{5.15}
\]

In the case of a point detector \( \eta = 1 \) but in general it depends on the experimental geometry of detection.

As soon as the coherence time \( \tau_c \) contains the information on the time scale evolution of the temporal correlation function, the coherence area \( A_c \) takes into account the spatial coherence of the light at the detector position. It is the width of the spatial coherence factor \( \mu(\vec{r}_1, \vec{r}_2) \) and it is defined by

\[
A_c = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\mu(\Delta x, \Delta y)|^2 d\Delta x d\Delta y, \tag{5.16}
\]

with \( \Delta x = x_1 - x_2 \) and \( \Delta y = y_1 - y_2 \), where \( \vec{r}_1 \equiv (x_1, y_1) \) and \( \vec{r}_2 \equiv (x_2, y_2) \) are two points on the detector. The area \( A_c \) is the area of a constructive interference zone between the fields diffused by different elements of the source at distance \( L \). It is the area of a speckle grain. For an homogeneous and incoherent light source with wavelength \( \lambda \) we can show that the coherence area is

\[
A_c = \frac{(\lambda L)^2}{A_{\text{source}}}, \tag{5.17}
\]

or \( A_c \simeq \lambda^2 / \Omega \), where \( \Omega \) is the solid angle under which the source is seen at the detector position.
Figure 5.3: Temporal contrast $\eta$ for the measure of intensity correlation for a square detector for different $N_c = A_{det}/A_c$.

The $\eta$ factor depends on the detector shape and it must be numerically calculated. We call $N_c = A_{det}/A_c$ the number of coherence areas contained in the detector surface. I report in figure 5.3 the $\eta$ factor for a square detector for different $N_c$ calculated from

$$\eta(N_c) = 4 \left[ \int_0^1 \frac{\sin(\pi \sqrt{N_c u})}{\pi \sqrt{N_c u}} (1 - u) du \right]^2,$$

(5.18)
as suggested by [36].

5.4 The construction of an ad-hoc detection apparatus

We have seen that in a measurement of the intensity correlation function it is fundamental to look at the light source within the spatial coherence of a single speckle grain or, at maximum, within a few grains, otherwise we lose all the information about the correlation (eq. 5.15). For example for the $\eta$ factor in the figure 5.3, for $N_c > 3$ we lose more than 80% of the contrast.

Regarding our cloud of cold atoms we have $A_{source} \simeq 0.8cm^2$ and $\lambda = 780nm$; we put the detector at a distance $L = 300mm$ far from the sample in the configuration (a) of the figure 5.4. We obtain from equation 5.17 a coherence area $A_c = 7 \cdot 10^{-5}mm^2$. That means that we have more than $10^3$ speckle grains in a detector with a 1mm diameter ($D_1$ of figure 5.4.
Thus in this detection condition we lose all the information about
the correlation: the light is completely incoherent and what we see is like
white noise. We cannot extract any information about the optical spectrum
because it is averaged out by the integration of a large number of speckle
onto the detector. Another inconvenience of this apparatus is that it is
very sensible to parasite light. We can select the spatial mode reducing the
diaphragm diameter ($D_1$ of figure 5.4 (a)) but we cannot avoid the parasite
light.

![Figure 5.4: Two different detection geometries. $D_1$: diaphragm for the
spatial mode selection. $D_2$: diaphragm for the definition of the effective
source (dotted lines).](image)

From here the need for a new geometry of detection. We built the
The idea is to use a lens and two diaphragms (see figure 5.4 (b)). With
the lens we create the image of the source in the position of the detector
with a certain magnification $g$; with the diaphragm $D_2$ we select an effective
surface of the source (dotted lines in the figure) and with the diaphragm $D_1$
we select the spatial mode (how many speckle we look at). The effective
size of the source is now $D_{\text{eff}} = D_2/g$, and it is this quantity, converted in
area, that enters in the calculation of the coherence area $A_c$ (eq. 5.17) and
thus of $\eta(A_{\text{det}}, A_{\text{source-\text{eff}}})$.

We are strongly constrained in choosing the dimensions requested ($L$
and the focal length $f$) by many factors like the space on the experimental
table and the presence of the vacuum cell around the atomic cloud, but also
by the possibility of seeing the image of the source by eye or by a camera.
If we do not put the detector in the precise position of the image formation,
the entire apparatus is useless because we have anyway a superposition of
different speckles. If the enlargement $g$ is too big we cannot see clearly
where the image is formed and so we cannot choose the right position for the detector. We chose $L = 150\text{mm}$, $f = 100\text{mm}$, so we obtained an enlargement $g = 2$ at the detector position. Then we put a diaphragm $D_2 = 100\mu\text{m}$ very close to the detector active surface. In these conditions the coherence area was $A_c = 7\text{mm}^2$ at the lens position, thus we needed a diaphragm $D_1 < 3\text{mm}$ close to the lens to select only a single speckle grain. The contrast predicted by figure 5.3 is $\eta = 0.6 - 1$. Furthermore, the parasite light from other diffusive objects before the lens is very strongly limited, and it is easy to build a black box to protect the detector from the parasite light coming from objects placed between the lens and the detector itself.

5.4.1 Calculation of the number of photons expected in one spatial coherence area. Study of two detectors.

In this section I report a calculation of the number of photons expected in one spatial coherence area in the case of a weak probe illuminating the atoms and in the case of the six MOT laser beams. We have already seen previously the scattering rate of a single atom in equation 1.10,

$$\Gamma_{sc} = \frac{\Gamma}{2} \frac{I/I_{sat}}{1 + 4\frac{\Delta^2}{\Gamma^2} + \frac{L}{L_{sat}}},$$

where $\Gamma = 6.06\text{MHz}$ for the $F = 3 \rightarrow F' = 4$ transition of $^{85}\text{Rb}$ (see figure 2.4), $I$ and $\Delta$ are respectively the intensity and the detuning of the incident laser beam. In the detection configuration of figure 5.4(b) we have an effective source surface $A_{eff} = \pi \frac{D_2^2}{2g} = 2 \cdot 10^{-5}\text{cm}^2$, that corresponds to an effective volume of about $V_{eff} \simeq 2 \cdot 10^{-5}\text{cm}^3$ if we consider a cylinder of 1 cm deep (the size of the cloud). A typical MOT has a density of $\rho \sim 10^{10}\text{at/cm}^3$, thus the effective number of atom that irradiate is $N_{at} = \rho V_{eff} \simeq 2 \cdot 10^5$. The scattering is isotropic so we have to consider also the solid angle $\Omega$ at which one atom sees the first diaphragm ($D_1 = 3\text{mm}$). The probability is given by the ratio $\frac{\Omega}{4\pi}$.

The total number of photons per second that arrive at the detector is thus given by

$$N_{ph} = \Gamma_{sc} \cdot \frac{\Omega}{4\pi} \cdot N_{at}(\text{ph/s}). \quad (5.19)$$

I consider a weak probe beam ($I = 0, 1I_{sat}$) shining the atoms at resonance $\Delta = 0$ or at the detuning of the Raman gain (I choose for example $\Delta = \Delta_R = 4, 5\Gamma$). I calculate the number of photons expected at the detector in the spatial coherence of a single speckle grain. The result gives $N_{ph} = 10^9\text{ph/s}$ at resonance and $N_{ph} = 1, 5 \cdot 10^4\text{ph/s}$ at $\Delta = \Delta_R$. In order to understand these numbers I remind that the same weak probe beam contains $10^{13}\text{ph/s}$. To detect such a weak number of photons we need a special detector. I used a photomultiplier $\text{MH972}$ produced by $\text{PerkinElmer}$. 

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With this detector I did not see anything for two reasons. The first reason is the relatively low quantum efficiency (Q.E.) of that photomultiplier: at 780nm Q.E. = 2% (data given by the constructor).

The second most important reason is the high dark current (D.C.) of such a detector: the constructor gives a value of $\text{D.C.} = 2 \times 10^3 \text{count/s}$ but experimentally I found a larger value $\text{D.C.} \sim 2 - 5 \times 10^4 \text{count/s}$. It is clear that such a detector couldn’t detect such a weak number of photons because the dark count are at least of the same order of the number of detected photons.

The random laser emission may occur at the gain frequency of the Raman structure, so at $\Delta \sim \Delta_R$, and the relevant signal will probably be weaker.

If we want to study the dynamics of the atoms in the MOT we can look at the light scattered from the six MOT beams. In our configuration they have a total power of about 300$mW$ and a very large beam waist ($\sim 2.5 - 3\text{cm}$). Thus the central intensity is $I \sim 20I_{sat}$. The MOT lasers are red-detuned by $3\Gamma$ from the atomic resonance. The equation 5.19 gives $N_{ph} = 5 \times 10^6 \text{ph/s}$ and also in this case there is a too low signal/noise ratio.

In order to have enough signal exceeding the dark noise we have to open the diaphragm $D_1$ and select more coherence area. If we take 10 coherence area we start to have a good signal (noise $\sim 5\%$). The counter part is that we lose the larger part of the information about correlation.

I made the same thing with an avalanche photodiode (APD). An APD is a photodiode with inverse polarization that starts to detect light above the breakdown voltage, that in our case is $V_{BD} = -157V$ (the negative sign indicate the inverse polarization). Above the breakdown voltage it is possible to increase the voltage up to the saturation ($V_s = -159V$). The APD could be in passive or active configuration depending of the electronic circuit around the photodiode. The practical difference is the different quenching time of the avalanche of electrons in the diode; in active configuration a feedback provides a very fast quench ($\sim 10\text{ns}$). Our detector is in passive configuration, having a quenching time of 2$\mu$s. The APD I used has not been designed to detect such a weak light signal in the regime of counting photons because of two reasons. The first one is the time of quench that limits the number of photons we can count up to $5 \times 10^5 / s$, the second one is the absence of cooling system for the diode. Indeed, the dark current depends strongly on the temperature. For our APD, at room temperature, the dark counts are $\text{D.C.} \sim 1 - 5 \times 10^5 \text{counts/s}$, depending on the voltage above the breakdown. Despite the higher quantum efficiency compared to the photomultiplier, the much higher noise due to the dark current prevents us to use it in our experiment.

The conclusion is the need for a new detector having lower and lower dark count (at least of the order of few tens of counts per second) and, if possible, a higher quantum efficiency.

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5.5 Conclusion

I made some measurement of the atomic fluorescence spectrum with a photomultiplier and with an avalanche photodiode. In the measurements in which I collect many speckles into the detector I observed the laser frequency noise amplified by the atoms. I could not observe anything in the spatial coherence of a single speckle grain because of the extreme weakness of the signal. I have verified that with the available detectors it is not possible to detect the interesting signal in one spatial coherence area. From the measurement of the intensity correlation and the optical power spectrum of the atomic fluorescence in one spatial coherence area it is possible, we argue, to have a signature of the future random laser emission.

The calculation reported above and the measurement done put in evidence the need for a less noisy and more sensible single photon detector. We have recently ordered such a detector associated with an auto-correlator that gives directly the intensity correlation function of the light.
Conclusion

In my thesis I have worked on the CAROL (Cold Atoms for a Random Optical Laser) ANR project at the “Institut Non Linéaire de Nice” in Sophia-Antipolis.

During my work I have played with a Magneto Optical Trap in almost all its components: laser beams, vacuum system, Rubidium atoms, magnetic field, temporal sequence and laser frequency controls. I have learned to cool down and to trap atoms in a MOT, and to measure the optical thickness of the obtained cold atoms clouds.

In my thesis, I addressed the question of how to combine gain and scattering in a cold atoms sample. I have studied the modification of the atomic response due to the pumping. I have especially implemented an experimental technique to measure the fluorescence from a weak probe in the background of a strong pump using a sort of synchronous detection. Thus, I have measured simultaneously the scattering rate and the transmission of a weak probe through the atomic sample, and developed an analysis to deduce the modification of the scattering cross section induced by the pump saturation and by the Raman gain structure. I have measured the characteristic lengths (scattering mean free path and linear gain length) of the pumped cold atoms sample, and with them I have predicted the random laser threshold for the Raman gain case.

I have also studied some possible detection technique for the future random laser realization; which are the heterodyne detection and the measurement of the intensity correlation from the noise power spectrum. I have studied and I have built an optical apparatus to detect the atomic fluorescence in the spatial coherence area of a single speckle grain, which also allows to reduce the parasite light. I have studied the properties of two detectors: a photomultiplier and an avalanche photodiode. I have demonstrated the need for a more sensible and less noisy detector for the future random laser detection.

The studies on the scattering properties of the pumped atoms have been an important step in the CAROL project. Even if the scattering is strongly reduced by the pumping, the Raman gain structure contributes to increase the scattering cross-section, in a way that can be explained by the calculation of the atomic polarizability (through the Kramers-Kronig relations).
Preliminary evaluations of the random lasing threshold have been made, and the understanding of the data I presented in this thesis will allow more systematic and accurate predictions.

A next step of the project will be to extent this study to the other possible gain mechanisms, so that we can choose the best one to realize a random laser. The theoretical part of this work is already under way. I also started to address the problem of the random laser detection, which is probably the more challenging issue of the projet. My study allows to evidence the requirements that an appropriate detection system should fullfill.
Appendix A

Clebsch-Gordan coefficients
Figure A.1: Clebsch-Gordan coefficients for the $F = 3 \rightarrow F' = 4$ transition of the $D2$ line of the $^{85}\text{Rb}$. 
Appendix B

Un laser aleatorio con atomi freddi: estrazione di informazioni dalla fluorescenza atomica

Un laser canonico è un dispositivo che realizza un equilibrio tra guadagno nel mezzo attivo e perdite dalla cavità ottica. Il ruolo primario degli specchi della cavità è quello di rimandare parte della radiazione nel mezzo di guadagno, consentendo un’amplificazione esponenziale della luce. Secondariamente la cavità seleziona i modi spaziali e temporali che vengono amplificati.

Un laser aleatorio è un laser in cui la retroazione nel mezzo attivo è data dalla diffusione multipla, e non dalla cavità. Diffusione multipla significa che un fotone, prima di riuscire ad uscire dal mezzo, effettua un cammino aleatorio nel mezzo causato da urti successivi con gli atomi. Ricordiamo che un evento diffusivo è sostanzialmente elastico se la radiazione incidente non satura gli atomi $s << 1$, dove il parametro di saturazione è $s = \frac{I}{I_{sat}} \frac{1+4\delta_{2}}{\Gamma_{2}}$ (vedi sezione 1.1.3). Una diffusione elastica conserva la frequenza e le relazioni di fase della luce (come una riflessione su uno specchio). Di conseguenza, anche la diffusione multipla seleziona i modi spaziali (speckle) e le frequenze dell’emissione del laser aleatorio.

Ci sono due regimi di retroazione: si parla di retroazione coerente quando la luce, in seguito a diffusione multipla, percorre dei percorsi chiusi (come piccole cavità); si parla di retroazione incoerente quando la retroazione serve unicamente per aumentare il tempo che un fotone impiega per lasciare il mezzo di guadagno. In entrambi i casi, se il guadagno è sufficientemente alto, si può arrivare ad una emissione laser isotropa in tutte le direzioni (aleatoria).
Letokhov nel 1968 [1] ha previsto questo fenomeno e ne ha calcolato la soglia di emissione (guadagno = perdite). Risolvendo l’equazione di diffusione con un termine di guadagno (eq. 1.1), egli è giunto alla seguente formula per la soglia

$$L_{cr} = 2\pi \sqrt{\frac{\ell_{sc} \ell_{g}}{3}}$$

(B.1)
dove $\ell_{sc}$ e $\ell_{g}$ sono rispettivamente il libero cammino medio tra una diffusione e l’altra e la lunghezza di guadagno, definita come la distanza che la luce deve percorrere per essere amplificata di un fattore $e$. Se il campione ha dimensioni maggiori della dimensione critica $L_{cr}$, il guadagno, che è proporzionale al volume del campione stesso, supera le perdite dalla superficie e il mezzo emette luce laser.


È noto da tempo che in un campione sufficientemente denso di atomi freddi la luce laser in risonanza con una transizione atomica è sottoposta a diffusione multipla prima di uscire dalla nube di atomi [4, 5]. Lo spessore ottico (vedi la sezione 2.3) è la quantità che caratterizza quanti eventi diffusivi avvengono mediamente nel campione. Negli ultimi anni sono stati fatti degli studi sui possibili meccanismi di guadagno che si possono indurre in un campione di atomi freddi grazie a tecniche di pompa-sonda [6]. Inoltre, sono stati realizzati dei laser con cavità aventi gli atomi freddi come mezzo attivo [7]. Nel capitolo 3 ho presentato i vari meccanismi di guadagno, prestando particolare attenzione al meccanismo di guadagno Raman che ho poi usato nelle misure.

Nel mio lavoro di tesi mi sono interessato a come unire il guadagno e la diffusione all’interno del campione di atomi freddi. Ho studiato come si modifica la risposta atomica in seguito al pompaggio. In particolare ho implementato una tecnica sperimentale per rivelare la fluorescenza di un debole fascio di sonda nel fondo della luce diffusa da una pompa molto più intensa. Ho modulato in intensità il fascio di sonda con una bassa frequenza e nello spettro di potenza della fluorescenza totale ho misurato l’altezza del
picco corrispondente alla frequenza di modulazione. Nel capitolo 4 ho spiegato più in dettaglio questa tecnica di misura. Usando questa tecnica ho misurato il tasso di diffusione e contemporaneamente la trasmissione di una debole sonda attraverso la nube di atomi freddi. Ho sviluppato un’analisi per dedurre la variazione della sezione d’urto atomica (sia di diffusione che di trasmissione) indotta dalla saturazione degli atomi, per la presenza della pompa, e dalla struttura del guadagno Raman. Ho in seguito misurato le lunghezze caratteristiche del campione ($\ell_{sc}$ e $\ell_g$) e calcolato, da queste, la soglia di emissione del laser aleatorio nel caso del guadagno Raman. Tutte queste misure con le relative spiegazioni dettagliate sono riportate nel capitolo 4.

Per quanto riguarda la rivelazione della luce emessa dal laser aleatorio è necessario fare alcune precisazioni. A differenza delle realizzazioni passate, nel caso degli atomi freddi la luce laser emessa sopra soglia ha una frequenza molto vicina a quella della luce proveniente dalla pompa ($\simeq 0,1\Gamma$ per il guadagno Raman). Per questo non è possibile filtrare la luce della pompa e osservare da sola la luce del laser aleatorio. Da qui la necessità di ideare una tecnica di rivelazione che permetta di distinguere le due frequenze. Anche questa parte è stata oggetto del mio studio. Un’idea è quella di misurare lo spettro ottico della luce diffusa tramite la tecnica di rivelazione eterodina; l’altra è quella di misurare direttamente la funzione di correlazione dell’intensità della luce diffusa, in quanto le sue proprietà cambiano per un laser sotto o sopra soglia. In entrambe le tecniche, però, è necessario rivelare unicamente un modo spaziale della luce di fluorescenza, altrimenti il battimento tra diversi modi sulla superficie del rivelatore annulla ogni correlazione e ogni altro battimento. Ho per questo costruito un apparato sperimentale che permetta di rivelare la fluorescenza atomica entro la coerenza spaziale di una singola speckle, e che permetta di minimizzare la luce parassita proveniente da altri oggetti diffusivi circostanti. Questo apparato sperimentale è riportato nel capitolo 5 insieme alle spiegazioni delle due tecniche di rivelazione. Inoltre ho studiato le caratteristiche di due rivelatori, un fotomoltiplicatore e un fotodiodo a valanga, per verificare la possibilità di un loro utilizzo nell’apparato di rivelazione. Ho concluso questo studio indicando la necessità di un rivelatore più sensibile (efficienza quantica maggiore) e meno rumoroso (minor numero di conteggi di buio).

In conclusione il mio lavoro ha apportato alcuni passi avanti nel progetto del random laser. In particolare ho constatato che, nonostante la diffusione sia fortemente ridotta dalla pompa, la struttura del guadagno Raman contribuisce ad aumentare la sezione d’urto di diffusione. Questo fenomeno è stato ben spiegato tramite l’utilizzo delle relazioni di Kramers-Kronig per calcolare la polarizzabilità atomica (vedi sezione 4.4). Inoltre è stata fatta una prima valutazione della soglia del laser aleatorio per il guadagno Raman.

Seguirà l’estensione dello studio fatto per il guadagno Raman agli altri meccanismi di guadagno al fine di trovare il meccanismo ottimale per realizz-
zare il laser aleatorio. La sfida più grande resta la rivelazione di un segnale significativo una volta raggiunta la soglia.
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