Self-Sustained Oscillations in a Large Magneto-Optical Trap

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We have observed self-sustained radial oscillations in a large magneto-optical trap, containing up to 10^{10} Rb^{85} atoms. This instability is due to the competition between the confining force of the magneto-optical trap and the repulsive interaction associated with multiple scattering of light inside the cold atomic cloud. A simple analytical model allows us to formulate a criterion for the instability threshold, in fair agreement with our observations. This criterion shows that large numbers of trapped atoms $N > 10^9$ are required to observe this unstable behavior.

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A large fraction of the stars in the upper Hertzsprung-Russell diagram present pulsations based on an interplay between modulated radiation pressure effects, which tends to increase the size of the star, and a collapse based on gravitational forces [1]. Instabilities also occur in other similar systems such as confined plasmas where a long-range Coulomb interaction has to be countered by a confining force to avoid an explosion of the plasma [2]. These systems are of fundamental importance for astrophysics and for controlled fusion and have thus been extensively studied in the past. However, it is either impossible (in the case of stars) or extremely difficult (in the case of confined plasmas) to perform experiments to study the full dynamics of such systems where collective effects play a dominant role. On the other side, allowing for adequate rescaling, alternative systems can present similar dynamics. A variety of interesting collective effects have thus been identified in charged colloidal systems [3]. Recently ultracold plasmas created by ionizing a cloud of laser cooled atoms became subject to increased attention [4]. Beyond the possibility of studying analogous effects as in astro and plasma physics, systems with long-range interactions are known to lead to nonextensive behavior and appropriate scaling laws are needed to predict macroscopic properties. Here we show that a large cloud of laser cooled atoms is an adequate system to study such collective effects. The radiation pressure of the multiply scattered photons in such clouds can indeed be related to a long-range Coulomb-type interaction [5]. We thus suggest an analogy between the dynamics of a large cloud of cold atoms, astrophysical systems, and plasma physics.

The effect of multiple scattering on the dynamics of the atoms is well known in the community of laser cooling of atoms, as multiple scattering has been a major limitation to obtain large phase space densities in cold atomic traps. Bose-Einstein condensation (BEC) in dilute atomic vapors has only been achieved after switching off all laser fields and using evaporation techniques [6]. More recently, multiple scattering of light in cold atoms has been used to study coherent light transport in random media [7]. This has led to an investigation of yet unexplored regimes, namely, the limit of very large number of cold atoms in the presence of quasiresonant light. Here we do not focus on the properties of the scattered light but on the mechanical effects of this light on the atoms. We have observed collective instabilities triggered by the repulsive interatomic force arising from multiple scattering, and identified a supercritical Hopf bifurcation separating the standard stable magneto-optical trap (MOT) operation from a yet undescribed unstable regime.

In order to estimate the relevance of plasma physics considerations to study multiple scattering of light by cold atoms it is worth deriving the equivalent of several plasma parameters for our system. The analogy with an $1/r^2$ repulsive Coulomb-type force [5] is obtained from evaluating the power scattered by one atom (P_{scatt}) and deriving the intensity I_2 incident on a second atom via $I_2 \propto P_{\mathrm{scatt}}/(4\pi r^2)$. The resulting radiation pressure force scales as $1/r^2$ and one can thus define an effective charge $\tilde{q}$ which depends on the absorption cross sections and laser intensity and is typically $\tilde{q} \approx 10^{-4}e$ [5]. A total interaction energy $\tilde{q}V = \frac{N\tilde{q}^2}{4\pi\epsilon_0 R}$ larger than the kinetic energy $k_B T$ of the particles leads to an increased diameter $L = 2R$ of the MOT when the number N of atoms exceeds $\approx 10^5$. Alternatively the Debye length $\lambda_D = \sqrt{\epsilon_0 k_B T/n\tilde{q}^2}$ above which collective effects become important is of the order of 100 μm, well below the typical size of a large MOT (several mm). Also, in our experiments the corresponding plasma frequency $\omega_D = \sqrt{n\tilde{q}^2/m\epsilon_0}$ is slightly larger ($\approx$200 Hz) than the relaxation rate of the atomic positions ($\approx$50 Hz). We thus expect our cloud to behave as a weakly damped plasma. Another interesting quantity is the ratio between the nearest neighbor Coulomb interaction and the kinetic energy $\Gamma_{\mathrm{Cb}} = \frac{\tilde{q}^2}{4\pi\epsilon_0 a}/k_B T$ with $a \simeq n^{-1/3}$ [8]. We estimate this quantity to be smaller than unity in our system, excluding thus any crystallization. An important aspect of these light induced collective interactions is that the effective charge $\tilde{q}$ depends on experimental control parameters, allowing for an engineering of the effective

charge which can be modified by orders of magnitude. Finally it might be possible to use the high phase space densities of a BEC and thus study strongly coupled plasma in the degenerate regime [9] as expected in neutron stars and white dwarfs [10].

Our cloud of cold atoms is confined in a MOT using laser-induced forces [11]. We collect Rb^{85} atoms from a dilute vapor using six large *independent* laser beams (beam waist 4 cm, power per beam $P = 30$ mW) thus avoiding the intensity imbalance and feedback mechanism responsible for the instability of Ref. [12]. Under standard operating conditions, the trapping lasers are detuned from the $F = 3 \rightarrow F' = 4$ transition of the $D2$ line by $\delta = -3\Gamma$ ($\Gamma/2\pi = 6$ MHz). A magnetic field gradient ($\nabla B \approx 10$ G/cm) is applied to generate a spatially dependent Zeeman shift yielding the restoring force of the trap. A repumping laser on the $F = 2 \rightarrow F' = 3$ of the $D2$ line is used to control the total number of atoms. We thus obtain a MOT with up to $N = 10^{10}$ atoms (diameter $L = 5$ mm, $T = 80$ μK) [13]. The size and shape of the cloud is monitored by imaging the MOTs fluorescence on a cooled CCD. The optical thickness b of the cloud at the trapping laser frequency is measured by a photodiode. To obtain a time-resolved information on the local density of the MOT, we also image a portion of the cloud on another photodiode.

Figure 1 illustrates the onset of spontaneous self-sustained oscillation for a sufficiently large number of atoms. We switched on the MOT at $t = 0$ and monitored the time evolution of the fluorescence from a portion of the MOT. This partial fluorescence signal is roughly describing the number of atoms in the observed region. Starting from $N = 0$ at $t = 0$, the trap fills with a time constant $\tau = 1.45$ s determined by the ambient Rb pressure. Below a critical number of atoms N_{th}, the size of the atom cloud increases with number of trapped atoms [14] but no specific dynamical behavior is observed. Above the threshold N_{th} the cloud switches to an unstable behavior characterized by periodic oscillations in the partial fluorescence signal. These radial oscillations of the cloud are self-sustained in the sense that no external modulation of any control parameter is present.

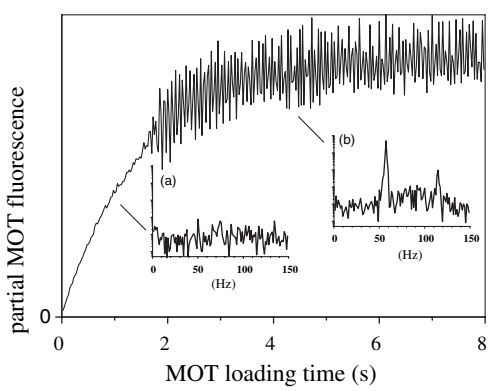

FIG. 1. Fluorescence of part of the MOT during a loading sequence. Below a critical number of atoms N_{th}, the size of the atom cloud increases without specific dynamical behavior. Above the threshold N_{th} the cloud switches to an unstable mode characterized by periodic oscillations in the partial fluorescence signal. Insets: Fourier transform of signal, with (a) a flat noise in the stable regime and (b) distinct oscillations in the unstable regime.

In the insets of Fig. 1 are shown Fourier transforms of the partial fluorescence. Below the instability threshold [inset (a)] a flat noise background is obtained. In contrast, in an unstable MOT, obtained for a larger number of atoms, distinct oscillation frequencies [inset (b)] appear, with higher harmonic components indicating the nonharmonic oscillation of the signal.

Indeed, the dynamics in the unstable regime can be more complex than a harmonic oscillation, as further illustrated in Fig. 2 where we detect the fluorescence from the center of the MOT. A high contrast modulation of the center fluorescence is observed in this experiment. We can speculate that the fast phase of decrease of the signal corresponds to a MOT expansion (decreased density at the center), whereas we associate the increasing part of the fluorescence to a slower compression phase. We observed that the precise shape of this oscillation depends on the laser beam alignment and on the monitored region of the MOT. However, the threshold separating the stable from the unstable regime was found to be very robust with respect to trap parameters.

Investigating the MOT at the instability threshold by varying 2 of the control parameters of the experiment (detuning, magnetic field gradient), we can map the phase diagram shown in Fig. 3 (full squares). The solid line corresponds to the theoretical prediction presented at the end of this Letter. As can be seen, the overall behavior is unstable when the trapping laser frequency is brought within roughly one natural width from resonance. This critical detuning depends here rather weakly on the mag-

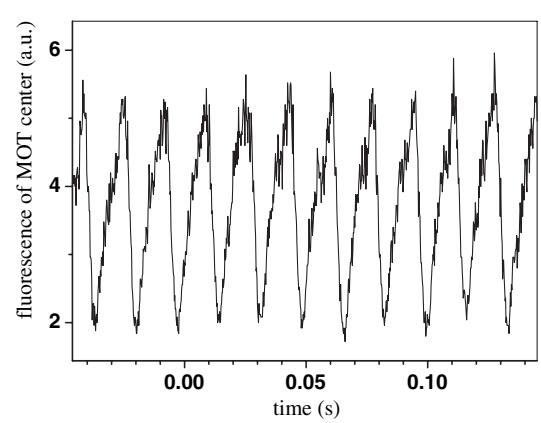

FIG. 2. Fluorescence of the MOT center. In the unstable regime, periodic oscillations appear in the absence of external modulations.

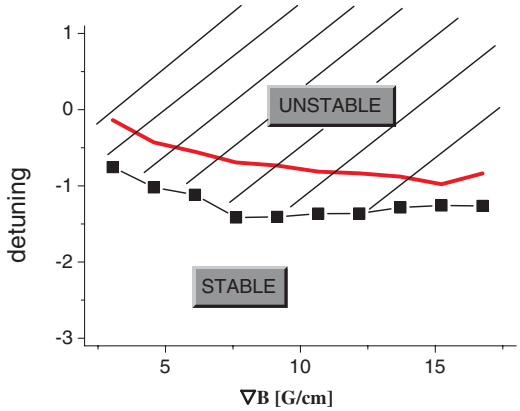

FIG. 3 (color online). Phase diagram of self-sustained oscillation: a $(\delta/\Gamma, \nabla B)$ cut in parameter space shows the separation between the stable (for larger detuning δ) and the unstable regime: experimental threshold values (squares) and 1-zone model prediction (solid line).

netic field gradient. However, the measured cloud size and number of atoms do vary quite a bit during this experiment (a factor 5 for N and a factor 2 for L). In addition, we systematically found an optical thickness $b \approx 1$ at the instability threshold. However, this is clearly not a sufficient condition for the onset of instabilities, since $b \approx 1$ is also observed in the stable region of Fig. 3.

We also carefully monitored the fluctuations of the total number of atoms when the MOT operates in the unstable regime, as, e.g., for the data in Fig. 2. We found these to be below 2%, as in the stable regime. This indicates that, for a given set of MOT parameters, the unstable cloud oscillates at fixed N.

To further characterize the transition to instability, we have checked that the amplitude of the oscillation continuously grows from zero as the control parameter (detuning or number of atoms in the experiments performed) crosses the threshold value. A Fourier analysis of the signal across the threshold showed that the instability starts at a nonzero frequency, which is closely related to the natural oscillation frequency of the harmonic trap. Furthermore, no hysteresis was observed despite explicit investigation. All these findings are consistent with a supercritical Hopf bifurcation.

Already in the stable regime, we observed some clear indications that strongly increasing the number of trapped atoms affects the way the MOT operates. As it is well known, the MOT inflates when atoms are added as a consequence of multiple scattering of light [5]. In addition to the standard $L \propto N^{1/3}$ law [5], we found for large number of atoms $N > 10^9$ a different scaling $L \propto N^{1/2}$ [14,15]. By monitoring the relaxation of the MOT after displacing it from its equilibrium position, we observed a crossover from an over-damped behavior at small N (typical for usual MOTs) to an under-damped behavior at large N. We interpret this finding as a consequence of the attenuation of the trapping beams inside the cloud, which reduces the friction at the center of the cloud. This could be envisioned as a precursor to the instability. Indeed, we found that just below the threshold (i.e., in stable operation), the MOT is systematically in the under-damped regime.

To explain the apparition of this new instability, we developed a simple model where the screened compression force of the MOT is competing against the repulsive interaction due to multiple scattering of light inside the cloud. We stress that this instability is thus qualitatively different from that studied in Ref. [12], where the use of retroreflected beams introduces the feedback necessary for the instability. The instability process of Ref. [12], which manifests as oscillations of the center-of-mass of the MOT, does not involve the long-range interatomic interactions which drives the behavior of our large MOT.

We propose in the following a very simple 1-zone model which exhibits an instability threshold. This model amounts to an extremely simplified mean field theory, based, however, on microscopic expressions for the light forces acting on the atoms. A more refined approach, beyond the scope of this Letter, could, e.g., involve hydrodynamical approximations [16].

We assume an homogeneous density and the size of the cloud L is related to the density n via the total number of atoms N: $n = N/L^3$. The dynamics along one symmetry axis (Ox) of a probe particle located outside of the cloud (at position $x > R = L/2$ from the trap center, with a velocity v) is then governed by the force:

$$F(x, v) = \frac{\hbar k\Gamma}{2} s_{\rm inc} \frac{e^{-b}}{1 + \frac{4(\delta - \mu x - kv)^2}{\Gamma^2}} - \frac{\hbar k\Gamma}{2} s_{\rm inc} \frac{1}{1 + \frac{4(\delta + \mu x + kv)^2}{\Gamma^2}} + \eta \frac{\hbar k\Gamma}{2} s_{\rm inc} \frac{1}{1 + \frac{4\delta^2}{\Gamma^2}} (1 - e^{-b}) \left(\frac{R}{x}\right)^2. \quad (1)$$

This expression relies on the low intensity Doppler model for the magneto-optical force (incident on-resonance saturation parameter $s_{\rm inc}$). The first term in this expression is the attenuated force of the laser passed through the cloud (with the corresponding Zeeman shift μx and Doppler shift kv); the second term corresponds to the nonattenuated force of the laser propagating in the opposite direction. In absence of the e^{-b} attenuation, these two terms give rise to the standard cooling (via the opposite Doppler terms kv) and trapping (via the opposite Zeeman terms μx) of cold atoms. The last term is the sum of all binary repulsive interactions which, using Gauss theorem, yields an $1/r^2$ repulsion for a probe particle outside the cloud. This term can be understood as the radiation pressure originating from the MOT with a total radiated power corresponding to attenuation of the 6 laser beams. Here η corresponds to the ratio between the absorption cross section of the inci-

dent laser frequency and the inelastically rescattered photons [5]. We now apply this model at the edge of the cloud ($x = R$). A linear stability analysis, using $x = R + \delta x e^{i\omega t}$ around the fixed point $F(R, v = 0) = 0$, yields the threshold condition for an instability ($\mathrm{Im}(\omega) = 0$):

$$C(\delta, \mu, b, R) = e^{-b} \frac{\delta - \mu R}{1 + \frac{4(\delta - \mu R)^2}{\Gamma^2}} + \frac{\delta + \mu R}{1 + \frac{4(\delta + \mu R)^2}{\Gamma^2}} = 0. \tag{2}$$

We find that for our experimental parameters the threshold is given with a good approximation by:

$$\delta + \mu R \approx 0. \tag{3}$$

It should be stressed that in standard MOTs one usually has $\mu R \ll |\delta|$: for $|\delta| = \Gamma$ and a magnetic field gradient $\nabla B = 10$ G/cm, condition (3) implies a MOT diameter of 8 mm. It is only with N in the 10^{10} range that such MOT sizes can be obtained. In this regime, the edge of the cloud is now exploring the nonlinear part of the magneto-optical force. It can be shown from the expression of the force that the threshold condition (3) corresponds to the passage from a positive to a negative friction at the edge of the cloud. Thus, a small velocity fluctuation is amplified instead of damped and the atoms at the edge are kicked away from the center of the cloud. The cloud thus expands until its optical thickness drops below a certain level, where the MOT is back to its standard mode of operation (weak repulsion) and the atoms are pushed back toward the center.

In order to further confront our simple model to the experiment, we computed the threshold value of the detuning δ using the control parameter ∇B and the measured values of the size of the cloud R and its optical thickness b at threshold. The result corresponds to the solid line in Fig. 3 and gives the correct order of magnitude and behavior for the instability threshold. This gives us further confidence about the qualitative validity of our model, as the size of the cloud varies by more than a factor 2 along the threshold boundary. Furthermore, both the threshold criterion (3) and the simple picture of the unstable dynamics near threshold are confirmed by recent results from a more involved model [17], which includes the inhomogeneous density distribution inside the cloud (N-zone model).

We have described in this Letter the observation of self-sustained oscillation in a large cloud of laser cooled atoms, arising from the competition between the MOT's confining force and the long-range multiple scattering repulsive interaction. This new instability process affects the behavior of large MOTs containing more than 10^9 atoms, a regime increasingly found, e.g., in experimental setups producing BECs. This observation shows that large clouds of cold atoms still present a rich dynamics with a variety of yet unexplored regimes. A simple 1-zone model has been presented which allows us to predict the instability threshold and to understand the underlying physical mechanism. Future possible investigations include the forced oscillation regime, the spectroscopy of excitation modes in this system, gas-liquid-crystal phase transitions in the degenerate regime, and feedback mechanisms allowing for stabilization of a large cloud of interacting particles. Progress on the theoretical aspects of the systems described in this Letter include exploiting mean field theory and molecular dynamic simulations. This should allow for a better understanding of the bifurcation observed in our experiment and lead to study statistical (thermodynamic) properties across the threshold. If the degenerate regime could be reached (or inducing similar interactions in a Bose-Einstein condensate) a mean field theory based on binary collisions as in usual Gross Pitaevskii equations will not be valid due to the long-range interaction, connecting this system to strongly correlated quantum systems.

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