Dissipation-Enhanced Collapse Singularity of a Nonlocal Fluid of Light in a Hot Atomic Vapor

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We study the out-of-equilibrium dynamics of a two-dimensional paraxial fluid of light using a near-resonant laser propagating through a hot atomic vapor. We observe a double shock-collapse instability: a shock (gradient catastrophe) for the velocity, as well as an annular collapse singularity for the density. We find experimental evidence that this instability results from the combined effect of the nonlocal photon-photon interaction and the linear photon losses. The theoretical analysis based on the method of characteristics reveals the main counterintuitive result that dissipation (photon losses) is responsible for an unexpected enhancement of the collapse instability. Detailed analytical modeling makes it possible to evaluate the nonlocality range of the interaction. The nonlocality is controlled by adjusting the atomic vapor temperature and is seen to increase dramatically when the atomic density becomes much larger than one atom per cubic wavelength. Interestingly, such a large range of the nonlocal photon-photon interaction has not been observed in an atomic vapor so far and its microscopic origin is currently unknown.

It has long been realized that light propagating in a nonlinear medium under the paraxial approximation can be interpreted in the context of quantum fluid dynamics [1–3]. Such phenomena in nonlinear Kerr-like media that we recently call fluids of light have been investigated in photorefractive crystals [4, 5], thermo-optic liquids [6, 7] and hot atomic vapors [8–11]. In this work, we use a hot atomic vapor to study the impact of dissipation and the nonlocal character of the interactions in the shock-wave dynamics [12]. The theory of dispersive shock waves has been developed for a long time [13], after pioneering investigations in the fields of tidal waves [14–16] and collisionless plasma [17–19]. It is only recently that dispersive shock waves have emerged as a general signature of singular fluid-type behavior [20–24] in areas as different as Bose-Einstein condensates [25–27], shallow-water [28], oceanography [29], plasma [30], viscous fluid conduits [31] and several optical systems, e.g. photorefractive crystals [32–34], passive cavities [35], and optical fibers [36–42]. An important ramification of the study of shock waves in the presence of a spatial nonlocal nonlinearity, a feature studied experimentally in liquids with strong thermo-optic effects [43–51]. In this context, shock-wave formation from a random speckled beam revealed that a highly nonlocal nonlinearity leads to a double shock-collapse singularity [50, 52]. Moreover, numerical simulations have shown the effect of nonlocal stabilization of nonlinear beams in a self-focusing atomic vapor [53]. In these works, the dissipation has been considered as a negligible perturbative effect.

Here, we report the observation of the double shock-collapse instability in a fluid of light in an atomic vapor, namely the shock singularity for the velocity (gradient phase of the field) and the annular collapse singularity for the density (field intensity). Our main result is to reveal a previously unrecognized and counterintuitive impact of the dissipation: The linear absorption of the optical field (i.e. the fluid of light) is shown to be responsible for an enhancement of the collapse instability. This unexpected result is enlightened by the theoretical analysis based on the method of characteristics for solving the hydrodynamic equations. Our work then also contributes to the development of the recent concept of “gain through losses” in nonlinear optics, where specific frequency-distributed losses can be imaged into spectral gain [54].

An important aspect of our experimental work is to unveil a strong nonlocal regime for fluids of light propagating in hot atomic vapors. Contrary to previous studies on atomic vapors [55–58] where interactions were seen to be local, we tuned the atomic density up to 20 atoms per cubic wavelength (by changing the temperature of the gas) which leads to an observable signature of nonlocality. By comparing our experimental data to numerical simulations we give an estimate of the range of the nonlocal interactions in our system. We propose a possible origin for such unexpected large value of nonlocality reported in this work.

Our fluid of light is a continuous-wave laser beam which propagates in a nonlinear Kerr-like medium along the z axis. Under the paraxial approximation, the dynamics of the slowly varying amplitude ψ of the laser
electric field is commonly described by the nonlinear Schrödinger (NLS) equation [1]. We consider the standard form of the nonlinear NLS equation:

\[
i\partial_z \psi = -\frac{\alpha}{2} \nabla^2 \psi - \frac{i}{2} \eta \psi + \gamma \int d\mathbf{r}' U(\mathbf{r} - \mathbf{r}') |\psi|^2(\mathbf{r}', z),
\]

where \(\nabla\) is defined in the transverse plane \(\mathbf{r} = (x, y)\) and the propagation axis \(z\) plays the role of an effective time. \(\alpha = 1/k_0\) is the dispersion (diffraction) parameter, where \(k_0\) denotes the wave number of the laser and plays the role of an effective mass. \(\eta\) quantifies the strength of the linear absorption in the medium. The interaction term includes an isotropic nonlinear response function \(U(\mathbf{r}) = U(|\mathbf{r}|)\) with the parameter \(\gamma = k_0 n_2\) that quantifies the strength of the interaction, \(n_2\) being the Kerr nonlinear index. We consider the repulsive photon-photon interactions (defocusing) regime \(\gamma > 0\). Without loss of generality, we consider the example of an exponential-shaped normalized response function \(U(\mathbf{r}) = (2\pi\sigma^2)^{-1} \exp(-|\mathbf{r}|^2/\sigma^2)\), but similar results are obtained with a Gaussian-shaped response \(U(\mathbf{r}) = (2\pi\sigma^2)^{-1} \exp(-|\mathbf{r}|^2/(2\sigma^2))\). Note that, in addition to atomic vapors [53], a nonlocal nonlinearity is found in several systems, e.g., dipolar Bose–Einstein condensates [59], nematic liquid crystals [60], glasses [61], liquids [51], or plasmas [62].

We use a \(L = 7\) cm long cell filled with a gaseous natural isotopic mixture of \(^{85}\text{Rb}\) and \(^{87}\text{Rb}\) as nonlinear medium. The fluid is created with a linearly polarized Ti:sapphire laser whose wavelength \(\lambda\) is tuned near the \(D_2\) resonance of \(^{87}\text{Rb}\). The laser detuning is adjusted from \(-14\) GHz to \(-2\) GHz with respect to the \(F = 2 \rightarrow F'\) transition of \(^{87}\text{Rb}\). In this range, the detuning is large compared to the Doppler broadening (\(\sim 250\) MHz) such that the Lorentzian shape of the line dominates. The atomic vapor density is controlled by adjusting and stabilizing the temperature of the cell from \(100^\circ\text{C}\) to \(160^\circ\text{C}\), which corresponds to a range of one \(^{87}\text{Rb}\) atom per \(\lambda^3\) to 20 atoms per \(\lambda^3\) respectively. With these two parameters (laser detuning and temperature), we can adjust the nonlinear Kerr index \(n_2\) and thus the effective photon-photon interaction \(\gamma\). To calibrate the value of \(n_2\), we measure the far field intensity of a collimated Gaussian beam with initial waist of 0.7 mm and peak intensity \(I = 6.5 \times 10^5 \text{ W m}^{-2}\). Due to self-phase modulation, this configuration generates concentric rings and provides a direct measurement of the nonlinear phase \(\phi_{NL} = k_0 n_2 IL\) accumulated by the beam along its propagation. In this work, we measure values of \(n_2\) from \(1 \times 10^{-11} \text{ m}^2 \text{W}^{-1}\) to \(3.8 \times 10^{-10} \text{ m}^2 \text{W}^{-1}\) [63].

The experimental setup is depicted in Fig. 1(a). We set the initial dimension of the fluid of light by demagnifying the beam down to a waist of \(w = 0.32\) mm. The waist is located at \(z = 0\) and the Rayleigh length of the beam exceeds the length of the cell. The total power can be adjusted from 0 to 1 W, leading to a peak intensity between 0 and \(I = 6.2 \times 10^6 \text{ W m}^{-2}\). In Fig. 1(b), we present a typical image of the cell output intensity compared to the input beam size. A clear difference between the initial and the final dimensions of the fluid is visible, due to the repulsive photon-photon interaction. Fig. 1(c) shows a typical radial cut of the intensity profile that we use to study the dynamics of the fluid.

The experimental protocol consists of fixing the laser transmission by adjusting the laser detuning \(\Delta\) after the cell has been heated to a given temperature \(T\). Then we explore the dynamics of the fluid of light by varying the beam power. The effective temporal evolution of the radial intensity profiles is shown in Fig. 2. Here we show data for a cell temperature of \(150^\circ\text{C}\) and a transmission of 40%. From each dataset (temperature and transmission) we can identify the development of a shock-wave. For early effective times, we observe a clear self-steepening of the shock-front, as expected for a conventional shock-wave [41]. Note that, in our experiment the shock-wave builds even in the absence of a background fluid to sustain it [37]. This is confirmed for late times where the wave breaking is regularized by the formation of the characteristic rapidly oscillating density front, which defines the shock point with power \(P_{\text{shock}}\). While the appearance of a shock is theoretically associated with a gradient catastrophe of the velocity, since our experimental observable is the density we identify \(P_{\text{shock}}\) as being the curve of Fig. 2 with the sharpest front and for which pre-oscillations emerge. These two criteria

![FIG. 1. (a) Simplified experimental setup. (b) Typical intensity profile obtained by imaging the output of the cell. The size of the beam at the entrance of the cell is illustrated by the white circle, whose radius equals the beam initial waist. Radial intensity profiles are extracted following the dashed line (\(r\)-axis). (c) Radial cut extracted from (b).](image_url)
are direct consequences on the density observable of the velocity gradient catastrophe. As expected, the typical spatial period of the oscillating front is given by the healing length $\Lambda = 1/\sqrt{2k_0 \gamma T_f}$ (where $I_f$ denotes the local intensity), with $\Lambda \simeq 50 \mu m$ in the case of Fig. 2.

At the shock point, the field exhibits a pronounced peak intensity on the shock-front annular profile. This observation is similar to that reported in liquids with strong thermo-optic effects [47, 50]. It is also apparently similar to the collective incoherent shock dynamics of speckle beams where the annular instability on the shock front was recognized as a collapse singularity [52]. The main result of our experiments is that such an annular peak intensity is enhanced (i) at high temperature (high nonlocality) and (ii) at low transmission (high dissipation), see Fig. 3. While the role of the vapor temperature can be understood by an increase of the nonlocality combined with dissipation has a dramatic impact. The nonlocality thanks to the vapor temperature, transmissions are chosen using the frequency detuning. The power at which the shock occurs is identified as explained through Fig. 2. (a) For $T = 130^\circ C$, the detunings at $T = 40\%$ and 20$\%$ are respectively $-3.3$ and $-2.1$ GHz. (b) For $T = 160^\circ C$, the detunings at $T = 40\%$ and 20$\%$ are respectively $-8.2$ and $-5.4$ GHz. Spatial and intensity profiles have been renormalized to improve the comparison of the collapse peak amplitude.

FIG. 2. Output radial intensity profile for different input powers. The cell is heated to 150$\,^\circ$C, the laser detuning is $\Delta = -5.5$ GHz and the transmission is $T = 40\%$. By looking at the different profiles we can identify the power corresponding to the shock point $P_{\text{shock}} = 300$mW. Note the formation of the rapidly oscillating density front regularizing the shock singularity with a spatial period given by the healing length $\Lambda$. Inset: Maximum intensity of the collapse peak as a function of the beam power. It hints at a faster-than-linear growth (resp. slower) below $P_{\text{shock}}$ (resp. beyond) in agreement with our theoretical analysis.

FIG. 3. Intensity profiles at the shock point showing that the collapse instability is enhanced by increasing either the dissipation, or the nonlocality (i.e. temperature). After fixing the nonlocality thanks to the vapor temperature, transmissions are chosen using the frequency detuning. The power at which the shock occurs is identified as explained through Fig. 2. (a) For $T = 130^\circ C$, the detunings at $T = 40\%$ and 20$\%$ are respectively $-3.3$ and $-2.1$ GHz. (b) For $T = 160^\circ C$, the detunings at $T = 40\%$ and 20$\%$ are respectively $-8.2$ and $-5.4$ GHz. Spatial and intensity profiles have been renormalized to improve the comparison of the collapse peak amplitude.

collapse singularity. We make the change of variable $\Psi(r, z) = \psi(r, z) \exp(i\eta z/2)$ in the NLS Eq.(1) and follow the usual procedure based on the Madelung transformation, $\Psi(r, z) = \sqrt{\rho(r, z)} \exp \left( i \phi(r, z) \right)$, to get coupled dispersive hydrodynamic equations for the evolutions of the density $\rho(r, z)$ and the velocity $\mathbf{u}(r, z) \propto \nabla \phi [32]$. In the experiment, the initial Gaussian beam is radially symmetric and this property is preserved by the dispersive hydrodynamic equations during the propagation of the beam. Accordingly, the velocity is radially outgoing $\mathbf{u} = u(r, z) \mathbf{r}/r$, where $r = |\mathbf{r}|$, and the density $\rho = \rho(r, z)$ is independent of the polar angle $\theta$. Considering the strongly nonlinear regime, the impact of dispersion effects can be neglected before the singularity and the hydrodynamic equations can be reduced to the effective one-dimensional radial system [64]:

\[
\partial_t \hat{\rho} + \alpha \partial_r (\hat{\rho} u) = 0, \tag{2}
\]

\[
\partial_t u + \alpha u \partial_r u + e^{-\gamma z} \partial_r V = 0, \tag{3}
\]

where $\hat{\rho}(r, z) = r \rho(r, z)$, and the effective nonlinear potential is given by $V(r, z) = \gamma \int_0^\infty \hat{U}(r, r') \hat{\rho}(r', z) dr'$, with $\hat{U}(r, r') = \frac{1}{2} \int_0^{2\pi} \hat{U}(\sqrt{r^2 + r'^2 - 2rr'\cos \theta}) d\theta$.

In the local limit without dissipation, Eq. (3) reduces to $\partial_t u + \alpha u \partial_r u + \gamma \partial_r \rho = 0$ and can be combined to (2) to develop an annular shock which is characterized by a divergence of $\partial_r u$ and $\partial_r \rho$, while $\rho$ remains finite and does not blow up. This scenario also holds in the weakly nonlocal regime $\sigma \ll \Lambda$. On the other hand, in the regime $\sigma > \Lambda$, the nonlocality changes the nature of the singularity. The main observation is that, at variance with $\partial_r \rho$ that diverges, the term $\partial_r V$ in (3) exhibits a regular behavior around the shock position. This regularization of $\partial_r V$ has a dramatic effect: in the absence of nonlo-
The development of the double shock collapse singularity is driven by the velocity gradient in Eq. (4). In the first term of (4), the amplitude of $\partial^{2}zV$ can be bounded by the maximal amplitude of $\hat{\rho}$ divided by $\sigma^{2}$. On the other hand, the second term in (4) describes a divergence of the form $|\xi| \sim 1/(z_{\infty} - z)$ close to the shock point $z_{\infty}$. Thus, there exists some propagation length beyond which the term $-\alpha \xi^{2}$ dominates $\partial^{2}zV(r, z)$, and $\xi(z)$ blows up in finite effective time $z$. Remarkably, we obtain the singular behaviors of $\xi(z)$ and $\hat{\rho}(z)$ just before $z = z_{\infty}$: $\partial_{z}u(R(z), z) \simeq -\alpha^{-1}(z_{\infty} - z)^{-1}$, $\hat{\rho}(R(z), z) \simeq \hat{\rho}(r_{0}, z_{0})(z_{\infty} - z_{0})/(z_{\infty} - z)$. Note the quantitative agreement obtained in Fig. 4(a)-(b) between the simulations of the NLS Eq. (1), the hydrodynamic Eq. (2-3), and the characteristic Eqs. (4-5). The singularity is not regularized by the dissipation (as for conventional diffusive shocks [12]), but by the dispersion effects neglected in Eqs. (2-3), which leads to the formation of the characteristic rapidly oscillating density front, as evidenced experimentally in Fig. 2.

This analysis unveils the unexpected role of dissipation. By quenching the first term in Eq. (4), dissipation $\eta$ favors the growth of $|\xi|$ described by the second term, which in turn favors the growth of $\hat{\rho}(z)$ before the singularity. This effect is illustrated in Fig. 4(d) where the dissipation is seen to strengthen the development of the collapse singularity at fixed nonlocality. Our experimental data, shown in Fig. 3, confirm this prediction since the amplitude of the collapse instability is enhanced by slightly decreasing the transmission from 40% to 20%. Similarly, our results show that nonlocality favors the development of the shock-collapse singularity by quenching the bounds of $\partial^{2}zV(r, z)$ in (4), see Fig. 4(c).

Using this model, we estimate the amount of nonlocality required to describe quantitatively our experimental results. We find a nonlocality of $\sigma \approx 70 \mu m$ for a temperature of $160^\circ C$. Such a large nonlocality coefficient is unexpected since, in hot atomic vapors, nonlocality is usually attributed to the ballistic transport of fast-moving excited atoms, which results in a nonlocal length scale of the order of $\sigma \approx 7.5 \mu m$ for $160^\circ C$ [53, 66, 67]. The larger nonlocal length scale needed to explain our observation requires to go beyond the simple two-level model used in [53] and includes more complex atom-light interaction, including for instance possible optical pumping [68] with long-range memory, the diffusive motion of atoms [67], collisional broadening [69] and other collective optical response mechanisms.

In conclusion, we have shown experimentally and theoretically that the linear dissipation combined to the nonlocal nonlinearity is responsible for a significant enhancement of the collapse instability. Furthermore, the theoretical analysis can be applied to a purely local non-linear...
interaction, which remarkably reveals that the shock formation (density gradient collapse) is associated with a density collapse only if there is dissipation. Our experimental platform enables the study of the crossover from a local nonlinear interaction to a nonlocal one by tuning the temperature of the vapor cell. We observed a nonlocal interaction which is an order of magnitude larger than previously reported for ballistic transport. These results open the way to tunable nonlocal physics with fluids of light and require multi-level atoms modelization to identify the microscopic mechanisms responsible for this effect.

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See supplementary material.

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