

COLD ATOMS AND MULTIPLE SCATTERING

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In this article we use a classical description of laser cooling of atoms. In a second part we describe the use of cold atoms for multiple scattering experiments and discuss some effects which appear for dense atomic media.

Laser cooling of atoms is usually described using a quantized atomic system (e.g. two-level system) and a classical description of the light. This is the so-called semi-classical description of laser cooling. It is also possible to quantize the light field, such as in the well known dressed states¹. In this paper however we want to use the opposite possibility and use the most classical description possible to understand laser cooling of atoms². As we will see, it will not be possible to eliminate completely the quantum effects for laser cooling of atoms: the atomic resonance is known to be dependant on Planck's constant and the recoil of an atom after absorbing or emitting one photon is also quantized. But given these two points, it is possible to compute the basic effects of the so-called Doppler cooling by a classical model. If one accepts the resonance frequency as a input parameter and a phenomenological diffusion constant for the residual heating, one could even apply this model to other situation such as e.g. acoustic waves. However it seems difficult to use this scheme to efficiently cool other systems than individual atoms, even if it is worthwhile noting that solid samples of matter have now been cooled using laser light³.

1 Classical model of Doppler cooling

Let us study the center of mass motion of atoms interacting with quasi resonant light. The radiative forces experienced by the atoms will depend on the detuning δ between the laser frequency ω_L and the atomic resonant frequency ω_{at} . If for example one wants to compute the radiation pressure one needs to know the scattering cross section, which, in the case of particles with an internal resonance, can be much larger than the geometrical size of the particle. In order to take into account these internal resonance effects, we will model the atom as a kernel surrounded by an elastically bound electron, with a resonance frequency ω_{at} . The laser light drives the electron and thus induces a dipole $\vec{d} = q\vec{r} = q(\vec{r}_e - \vec{R})$ which, in the driven regime, oscillates



at the driving frequency ω_L . It will be the interaction between this driven dipole and the electro-magnetic field of the laser which acts on the center of mass of the atom.

We will hence proceed in two steps: first compute the dipole induced by the laser light and second study the motion of this oscillating dipole with the electromagnetic field.

1.1 Internal motion: elastically bound electron

In this model we will suppose that the distance $\vec{r} = \vec{r}_e - \vec{R}$ between the electron and the kernel of the atom follows the equation²

$$\boxed{\frac{d^2}{dt^2} \vec{r} + \Gamma \frac{d}{dt} \vec{r} + \omega_{at}^2 \vec{r} = \frac{\vec{f}_{ext}}{m_e}} \quad (1)$$

The total force acting on the electron is composed by the force \vec{f}_E due to the electric field at the position $\vec{r}_e = \vec{R} + \vec{r}$ of the electron:

$$\vec{f}_E = q \vec{E}(\vec{r}_e, t)$$

and by a component due to the magnetic field:

$$\vec{f}_B = q \frac{d\vec{r}_e}{dt} \wedge \vec{B}(\vec{r}_e, t)$$

The ratio between the amplitude of these two forces is of the order of

$$\frac{f_B}{f_E} \simeq \frac{dr_e}{dt} \frac{1}{c} = \frac{v_e}{c} \ll 1$$

and we hence can neglect the effect of the magnetic field for computing the relative motion of the electron. Furthermore, the mass of the kernel being much larger than that of the electron, the distance $\vec{r} = \vec{r}_e - \vec{R}$ between the electron and the kernel of the atom is determined by the motion of the electron. We will use the complex notation for the electric field for a monochromatic linear polarized light:

$$\vec{E}(\vec{r}, t) = E_0(\vec{r}) \vec{e}_x \exp(-i\omega_L t) \quad (2)$$

Using eqs. (1) and (2) and only taking into account the electric field force on gets a driven solution $\vec{r}(t) = \vec{r}_0 \exp(-i\omega_L t)$ with:

$$-\omega_L^2 \vec{r}_0 - i\omega_L \Gamma \vec{r}_0 + \omega_{at}^2 \vec{r}_0 = \frac{q E_0(\vec{r}_e)}{m_e} \vec{e}_x$$

Defining the polarizability $\alpha(\omega_L)$ of the atomic dipole by:

$$\vec{d} = q \vec{r} = \varepsilon_0 \alpha(\omega_L) \vec{E}$$

one thus obtains:

$$\alpha(\omega_L) = \frac{1}{(\omega_{at}^2 - \omega_L^2 - i\omega_L\Gamma)} \frac{q^2}{\epsilon_o m_e}$$

We will use the real and the complex part of $\alpha(\omega_L)$: $\alpha = \alpha' + i\alpha''$:

$$\begin{aligned} \alpha' &= \frac{\omega_{at}^2 - \omega_L^2}{(\omega_{at}^2 - \omega_L^2)^2 + (\omega_L\Gamma)^2} \frac{q^2}{\epsilon_o m_e} \\ \alpha'' &= \frac{\omega_L\Gamma}{(\omega_{at}^2 - \omega_L^2)^2 + (\omega_L\Gamma)^2} \frac{q^2}{\epsilon_o m_e} \end{aligned} \quad (3)$$

With real notations for the electric field and for the dipole \vec{D} one thus has

$$\vec{D} = \text{Re } \vec{d} = \text{Re} \left[\epsilon_o \alpha(\omega_L) \vec{E} \right]$$

For a wave propagating along Oz such as:

$$\vec{E}(\vec{r}, t) = E_o(\vec{r}) \vec{e}_x \exp[-i(\omega_L t - kz)]$$

one gets:

$$\vec{D} = \epsilon_o |\alpha| E_o(\vec{r}) \vec{e}_x \cos[-(\omega_L t - kz) + \varphi_\alpha]$$

where $\alpha = |\alpha| \exp(i\varphi_\alpha)$. The induced dipole follows the driving electric field with some delay. This delay depend on the detuning between the laser frequency and the resonance frequency of the dipole. If the electric field oscillates very slowly compared to the dipole resonance frequency ($\omega_L \ll \omega_{at}$), we have $\alpha' \gg \alpha''$ and the induced dipole almost immediately follows the electric field excitation with a static polarizability:

$$\alpha_{stat} = \frac{q^2}{\epsilon_o m_e \omega_{at}^2} = \alpha_0$$

For a resonant excitation ($\omega_L = \omega_{at}$) we have $\alpha' = 0$, i.e. a purely imaginary polarizability, and the dipole is in quadrature phase with the driving field. Defining the detuning $\delta = \omega_L - \omega_{at}$ as the difference between the laser frequency ω_L and the atomic resonance frequency ω_{at} one gets for a quasi-resonant excitation ($\delta \ll \omega_L$, $\omega_L \simeq \omega_{at}$) (figure 1):

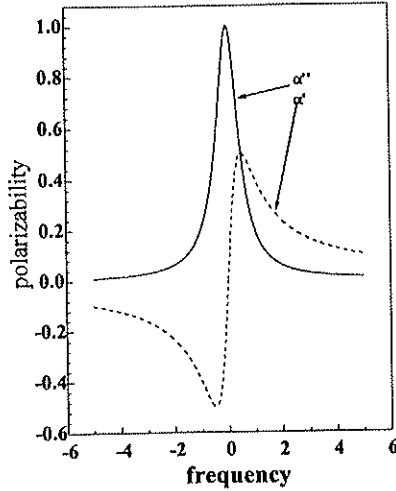


Figure 1. Atomic polarisability $\alpha[\alpha_0\omega_L/\Gamma]$: real part α' (dashed line) and imaginary part α'' (solid line) as a function of detuning $\omega_L - \omega_{at}[\Gamma]$

$$\alpha' = \frac{-\delta\omega_L}{\delta^2 + \frac{\Gamma^2}{4}} \frac{\alpha_0}{2} \quad (4)$$

$$\alpha'' = \frac{\frac{\Gamma}{2}\omega_L}{\delta^2 + \frac{\Gamma^2}{4}} \frac{\alpha_0}{2} \quad (5)$$

Remarks i) In order to compute this polarizability we have approached the electric field $E_0(\vec{r}_e)$ at the position of the electron with the field at \vec{R} of the center of mass M of the atom. This approximation is valid if the distance between the electron and the center of mass of the atom is small compared to the scale on which the electric field varies, i.e. small compared to the wavelength of the laser: $|\vec{r}| = |\vec{r}_e - \vec{R}| \ll \lambda$. This approximation is called electric dipole approximation as one can consider the atom as a point dipole on the scale of the wavelength λ . Note that even in this dipole approximation, real atoms have a more complex internal structure (Zeeman sublevels e.g.) and exhibit some features which cannot be described by a classical dipole oscillation.

ii) The damping of the dipole can be explained by the radiation of the oscillating dipole. This radiation depends on the frequency of the oscillation and strictly speaking one should replace Γ by $\Gamma \frac{\omega^2}{\omega_{at}^2}$. But we will only be interested

in frequencies close to the resonant frequency and we will thus neglect the change of the damping on the scale of δ . \square

1.2 Radiation forces acting on the atom: "Classical approach"

The force acting on the center of mass of atom, considered now as an oscillating dipole, has two components: one due to the electric field and one due to the magnetic field of the incident laser field, propagating along the Oz axes. The force \vec{f}_E due to the electric field, which we take polarized along the Ox axes:

$$\vec{E}(\vec{r}, t) = E_0(\vec{r})\vec{e}_x \exp(-i\omega_L t)$$

is directed parallel to this electric field:

$$\vec{f}_E = \sum q\vec{E}(\vec{r}, t) \propto \vec{e}_x$$

The magnetic force \vec{f}_B on the contrary will be directed along the axes of propagation of the laser (for a linear polarized plane wave)! The electron driven by the electric field has a velocity along the axes Ox : $\frac{d\vec{r}_e}{dt} = \left| \frac{d\vec{r}_e}{dt} \right| \vec{e}_x$ and for a magnetic field along Oy : $\vec{B}(\vec{r}_e, t) = B_0(\vec{r}_e)\vec{e}_y \exp(-i\omega_L t)$ one gets:

$$\vec{f}_B = q \frac{d\vec{r}_e}{dt} \wedge \vec{B}(\vec{r}_e, t) = q \left| \frac{d\vec{r}_e}{dt} \right| B_0(\vec{r}_e) \exp(-i\omega_L t) \vec{e}_z$$

It is thus clear that one cannot neglect the effect of the magnetic field for computing the force acting on the center of mass of the atom! One can keep in mind the model of the electron driven by the electric field and it is the magnetic force which acts on this moving charged particle. Although this is not a complete description, it allows one e.g. to understand why the radiation pressure force is along the axes of propagation of the laser light, without using a quantum treatment for the electric field of the laser.

Despite $\frac{f_B}{f_E} \simeq \frac{v}{c} \ll 1$ one cannot neglect the magnetic force when evaluating the force acting on the center of mass of the atom. The magnetic and the electric field indeed yield forces which are not directed along the same axes. For a plane wave polarized along the Ox axes propagating along Oz the magnetic force seems to give the radiation pressure. It is the sum of the electric and magnetic forces that give rise to the radiation pressure force acting on atoms⁴.

The electric field will yield a force on both charged parts of the atom: the electron and the nucleus:

$$\vec{f}_E = |q| \vec{E}(\vec{R}, t) - |q| \vec{E}(\vec{r}_e, t)$$

The two components of the electric force have opposite signs. However the net electric force is not zero as the electric field is not the same at the location \vec{r}_e of the electron and \vec{R} of the nucleus. The electric force is hence a differential effect.

The magnetic force on the other hand depends on the velocities of the charged particles. As the electron does move much faster than the nucleus, one only needs to consider the magnetic force acting on the electron. But one has to consider this force together with the electric force acting on both charges.

Electric force The electric force can be evaluated by making a first order expansion of $\vec{E}(\vec{r}_e, t)$:

$$\vec{E}(\vec{r}_e, t) \simeq \vec{E}(\vec{R}, t) + \left\{ \left[(\vec{r}_e - \vec{R}) \cdot \overrightarrow{grad} \right] \vec{E}(\vec{r}_e, t) \right\}_{\vec{r}_e = \vec{R}}$$

Taking $\vec{d} = q(\vec{r}_e - \vec{R})$ the electric force acting on the atom is:

$$\vec{f}_E = \left\{ \left[\vec{d} \cdot \overrightarrow{grad} \right] \vec{E}(\vec{r}_e, t) \right\}_{\vec{r}_e = \vec{R}}$$

with components along each axes \vec{e}_i ($i = x, y, z$):

$$f_{E,i} = \left\{ \left[\sum_j d_j \frac{\partial}{\partial x_j} \right] E_i(\vec{r}_e, t) \right\}_{\vec{r}_e = \vec{R}}$$

The spatial scale of variation for $\vec{E}(\vec{r}_e, t)$ is the wavelength λ . The electric force would then be zero if the electric field were uniform and its effect on the atomic dipole only appears at the first order in $\frac{|\vec{r}_e - \vec{R}|}{\lambda}$. We now have computed the instantaneous force \vec{f}_E which needs to be averaged over the fast optical frequency in order to describe the slow motion of the center of mass of the atom.

Magnetic force For the magnetic force we restrict ourselves to:

$$\vec{f}_B = q \frac{d\vec{r}_e}{dt} \wedge \vec{B}(\vec{r}_e, t)$$

and at the lowest order one has:

$$\vec{f}_B = q \frac{d\vec{r}}{dt} \wedge \vec{B}(\vec{R}, t)$$

As $\vec{d} = q\vec{r} = \varepsilon_0 \alpha(\omega_L)\vec{E}$ one can write:

$$\vec{f}_B = \frac{d}{dt} \left(\vec{d} \wedge \vec{B}(\vec{R}, t) \right) - \vec{d} \wedge \frac{d}{dt} \vec{B}(\vec{R}, t)$$

Taking $\frac{d}{dt} \vec{B}(\vec{R}, t) \simeq \frac{\partial}{\partial t} \vec{B}(\vec{R}, t)$ (the velocities of the charges are small compared to c) and using Maxwell's equation $\frac{\partial \vec{B}}{\partial t} = -\text{rot} \vec{E}$ one can express the magnetic force as a function of the electric field:

$$\vec{f}_B = \frac{d}{dt} \left(\vec{d} \wedge \vec{B}(\vec{R}, t) \right) + \vec{d} \wedge \text{rot} \vec{E}$$

or, for the \vec{e}_i component:

$$f_{B,i} = \frac{d}{dt} \left(\vec{d} \wedge \vec{B}(\vec{R}, t) \right)_i + \sum_j \left(d_j \frac{\partial}{\partial x_i} E_j - d_j \frac{\partial}{\partial x_j} E_i \right)$$

The time average of the first term is zero and we hence neglect this part in the following.

Total force The total average force $\langle \vec{f} \rangle$ on a atom by a light wave is:

$$\langle \vec{f} \rangle = \langle \vec{f}_E \rangle + \langle \vec{f}_B \rangle = \left\{ \left[\vec{d} \cdot \text{grad} \right] \vec{E}(\vec{r}_e, t) \right\}_{\vec{r}_e = \vec{R}} + \vec{d} \wedge \text{rot} \vec{E}$$

with components along \vec{e}_i :

$$\langle f_i \rangle = \left\{ \left[\sum_j d_j \frac{\partial}{\partial x_j} \right] E_i(\vec{r}_e, t) \right\}_{\vec{r}_e = \vec{R}} + \left\{ \sum_j \left(d_j \frac{\partial}{\partial x_i} E_j - d_j \frac{\partial}{\partial x_j} E_i \right) \right\}_{\vec{r}_e = \vec{R}}$$

$$\langle f_i \rangle = \left\{ \sum_j d_j \frac{\partial}{\partial x_i} E_j \right\}_{\vec{r}_e = \vec{R}}$$

This force seems to derive from a time average potential:

$$W = -\langle \vec{d} \cdot \vec{E} \rangle = -\left\langle \sum_j d_j E_j \right\rangle$$

where the gradient is taken only on the electric field \vec{E} and not on the dipole \vec{d} :

$$\langle \vec{F} \rangle = -\overrightarrow{\text{grad}} W = \left\langle \sum_j d_j \overrightarrow{\text{grad}} E_j \right\rangle$$

Average radiation force Let us consider the following electric field $\vec{E}_0(\vec{r}) = \vec{e}_x E_0(\vec{r}) \exp(ikz - i\omega_L t)$, with $E_0(\vec{r})$ real.

Returning to real notations for the fields and the dipoles one has:

$$\vec{D} = \text{Re } \vec{d} = \text{Re} \left(\varepsilon_0 \alpha \vec{E}_0(\vec{r}) \exp(-i\omega_L t) \right)$$

For a wave propagating along Oz one gets: $\vec{E}_0(\vec{r}) = \vec{e}_x |\vec{E}_0(\vec{r})| \exp(ikz)$ and

$$\vec{D} = \varepsilon_0 |\vec{E}_0(\vec{r})| \vec{e}_x \left(\alpha' \cos(\omega_L t - kz) + \alpha'' \sin(\omega_L t - kz) \right)$$

To calculate the average force let's first take

$$\overrightarrow{\text{grad}} \text{Re} [E_0(\vec{r})(-i\omega_L t)] = \text{Re} \left\{ \overrightarrow{\text{grad}} [|E_0(\vec{r})| \exp(ikz)] (-i\omega_L t) \right\}$$

$$\begin{aligned} \overrightarrow{\text{grad}} \text{Re} [E_0(\vec{r})(-i\omega_L t)] &= \overrightarrow{\text{grad}} [|E_0(\vec{r})|] \cos(\omega_L t - kz) \\ &\quad + k \vec{e}_z |E_0(\vec{r})| \sin(\omega_L t - kz) \end{aligned}$$

We then obtain the instantaneous force:

$$\begin{aligned} \vec{F} &= \left[\varepsilon_0 |E_0(\vec{r})| \left(\alpha' \cos(\omega_L t - kz) + \alpha'' \sin(\omega_L t - kz) \right) \right] * \\ &\quad \left[\overrightarrow{\text{grad}} [|E_0(\vec{r})|] \cos(\omega_L t - kz) + k \vec{e}_z |E_0(\vec{r})| \sin(\omega_L t - kz) \right] \end{aligned}$$

The time average force is thus

$$\langle \vec{F} \rangle = \varepsilon_0 \left(\frac{1}{4} \alpha' \overrightarrow{\text{grad}} [|E_0(\vec{r})|^2] + \frac{1}{2} \alpha'' k \vec{e}_z |E_0(\vec{r})|^2 \right) \quad (6)$$

1.3 Resonant radiation pressure

The second term of (6) is called the resonant radiation pressure \vec{f}_{rad} . It is aligned along the direction of propagation of the laser (\vec{e}_z) and it is proportional to the laser intensity: $I_{inc} = \frac{1}{2}\epsilon_0 c \left| \vec{E} \right|^2$:

$$\vec{f}_{rad} = \alpha'' k \vec{e}_z \frac{I_{inc}}{c} \quad (7)$$

One can thus define a scattering cross section σ_{at} for the atoms. By taking: $\vec{f}_{rad} = \alpha'' k \vec{e}_z \frac{I_{inc}}{c} = \sigma_{at} \frac{I_{inc}}{c} \vec{e}_z$ one gets: $\sigma_{at} = \alpha'' k$, which depends on the detuning δ (figure 2):

$$\sigma_{at}(\delta) = \frac{\frac{\Gamma}{2}}{\delta^2 + \frac{\Gamma^2}{4}} \frac{q^2}{2\epsilon_0 c m_e} \quad (8)$$

At resonance one has $\sigma_{at}^{res} = \alpha_0 k \frac{\omega_{at}}{\Gamma}$. Taking for the damping constant the radiation losses due to the oscillating electron ($\Gamma = \frac{4}{6mc^3} \frac{q^2}{4\pi\epsilon_0} \omega_L^2 \simeq \frac{4}{6mc^3} \frac{q^2}{4\pi\epsilon_0} \omega_{at}^2$), one finds at resonance:

$$\sigma_{at}^{res} = \frac{3\lambda_{at}^2}{2\pi} \quad (9)$$

Remarks "Comets": this radiation pressure is responsible for the neural tail of comets. In these tails small particles (of diameter less than 1 micron) are pushed away from the sun. The radiation pressure which scales as surface/distance² dominates for small particles over the gravitational attraction which scales as volume/distance².

"Quantum approach": It is possible to evaluate the radiation pressure force of a plane monochromatic wave acting on an atom by a linear momentum conservation argument. As a classical electric field is not a eigenstate of the momentum operator, we use a quantum description of the light field in terms of photons. For a wave propagating along the Oz axes, each absorption process give rise to a momentum transfer of $\Delta \vec{p} = \hbar k \vec{e}_z$. The emission of photons will occur in a random direction such that, on average, the momentum transfer after several fluorescence cycles will be zero for the emission processes. One thus gets an average momentum transfer of $\langle \Delta \vec{p} \rangle = \hbar k \vec{e}_z$ per fluorescence cycle. The average force \vec{f}_{av} , depending on the number of fluorescence cycles per second $\frac{dN}{dt}$ (hence also on the laser intensity and detuning) is thus given

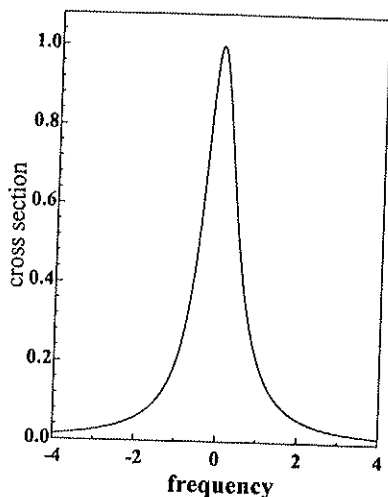


Figure 2. Cross section σ/σ_{res} as a function of detuning δ/Γ

by:

$$\vec{f}_{av} = \frac{dN}{dt} \hbar k \vec{e}_z$$

directed along the axes of propagation of the incident laser light. This argument is based on a quantum treatment of the laser light (the electric field being quantized in terms of photons) and is not along the main line of the calculation followed in this article. However it clearly shows that the direction of the force acting on the atom can be along the direction of propagation of the laser light and hence transverse to the electric field! A similar argument could give the force acting on a mirror when reflecting light.

It is possible to reconcile the "classical" and "quantum" description of

the radiation pressure by rewriting the force (7) as:

$$\vec{f}_{rad} = \hbar \vec{k} \frac{\Gamma I_{inc}}{2 I_{sat}} \frac{\Gamma^2/4}{\delta^2 + \Gamma^2/4} \quad (10)$$

with $I_{sat} = \frac{\Gamma^2 \hbar c}{2 \alpha_o \omega_L} = \frac{\Gamma^2 \hbar c \epsilon_o m_e \omega_L}{2 q^2}$. In the case of Rubidium atoms one gets for example, $I_{sat} = 1.6 mW/cm^2$. This expression (10) allows for a simple physical explanation of the radiation pressure force. During each fluorescence cycle one has a $\hbar \vec{k}$ transfer of momentum from the laser field to the atom. The time scale for one fluorescence cycle depends on the lifetime $\frac{1}{\Gamma}$ of the excited state of the atom and on the laser intensity needed to reexcite the atoms after the spontaneous emission. The number $\frac{dN}{dt}$ of absorbed and reemitted photons per unit time is:

$$\frac{dN}{dt} = \frac{\Gamma I_{inc}}{2 I_{sat}} \frac{\Gamma^2/4}{\delta^2 + \Gamma^2/4}$$

which has the resonant frequency dependence. □

1.4 Dipole force

The first term of (6) is called dipole force \vec{f}_{dip} . A particle with an real polarizability α' is attracted to a spatial region where its potential energy $W = -\vec{d} \cdot \vec{E}$ is minimum. With $d = \epsilon_o \alpha' \vec{E}$ one has:

$$W = -\epsilon_o \alpha' \left| \vec{E} \right|^2$$

One thus obtains a force oriented towards regions of high electric field in the case of $\alpha' > 0$ (high field seekers, such as dielectric spheres in air) and towards low electric field for $\alpha' < 0$ (low field seekers, such as air bubbles in champagne).

Remark Whereas the radiation pressure force can be explained by fluorescence cycles of absorption followed by spontaneous emission, the dipole force can be expressed in a quantum approach in terms of absorption followed by stimulated emission processes. □

Let us consider the case of a standing wave obtained by two counter-propagating plane waves $\left| \vec{E} \right|^2 = 4 \left| \vec{E}_0 \right|^2 \cos^2(kz)$. The dipole force for an atom located at z is in this case:

$$\vec{f}_{dip} = -8 \hbar \vec{k} \delta \frac{I_{inc}}{I_{sat}} \frac{\Gamma^2/4}{\delta^2 + \Gamma^2/4} \sin(2kz)$$

where I_{inc} is the incident intensity of each of two plane waves producing the standing wave. Note that this force changes sign when moving along Oz .

Order of magnitude: For an on-resonant laser ($\delta = 0$) with an intensity of $I = 1 \text{ mW/cm}^2$ the radiation pressure force is $f_{rad} = \alpha'' k \frac{I}{c} \simeq 10^{-20} \text{ N}$. This force is 10^4 times larger than the gravitational force: $f_g = Mg = 10^{-24} \text{ N}$! Even though each momentum transfer is quite small the radiation pressure forces are huge because after the emission of photons the atoms can quickly be reexcited to the upper state and up to 10^7 cycles of fluorescence per second can be achieved. One condition for this to happen is that, after a spontaneous emission, the atoms fall back into the initial ground state from where it quickly can be reexcited (so-called closed transitions). In the elastically bound electron model, this condition does not appear explicitly, except for the fact that we suppose one single atomic frequency to be present. In some atoms this condition can be fulfilled (sometimes at the expense of an additional "repumping" laser), but in the case of molecules, it is much more difficult to find closed transitions useful for laser cooling.

1.5 Doppler cooling

We now apply the radiation pressure forces to study laser cooling of atoms, i.e. in order to reduce the width of the velocity distribution of a sample of atoms. The simplest idea for such a cooling has been proposed in 1975 by T. Hänsch and A. Shawlow⁵ for neutral atoms and by D. Wineland and H. Dehmelt⁶ for ions. Consider the case of two laser counterpropagating along Oz with the same frequency ω_L . This argument can be generalized to three dimensions, but for simplicity we will restrict ourselves to one dimension. An atom interacting with such a laser configuration will be submitted to the radiation pressure forces calculated in section 1.2. A detailed analysis of this situation has to include both effects of the resonant radiation pressure and of the dipole force. But a basic explanation of the so-called Doppler cooling can be given by simply adding independently the resonant radiation pressure forces of the two propagating laser fields.

Let us consider the case of the laser frequency being detuning below the atomic resonant frequency: $\delta = \omega_L - \omega_{at} < 0$ ("red detuning"). For an atom at rest ($v_z = 0$), the excitation by the laser light will not be efficient as the resonance condition for neither laser will be fulfilled. If now an atom is moving ($v_z \neq 0$) it will experience different frequencies from the two laser fields. In the atom's rest frame, one of the laser frequencies will be shifted towards higher frequencies, the other one towards lower frequencies. For a "red detuning" ($\delta < 0$) the atom will shift into resonance ($\delta - k_z v_z = 0$) with

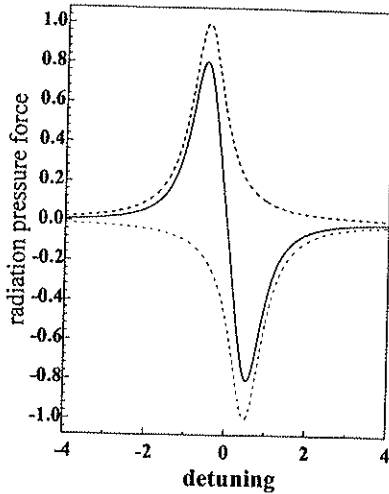


Figure 3. Radiation pressure force as a function of detuning δ/Γ

the laser propagating opposite to the atoms velocity: $\delta = k_z v_z < 0$. The other laser on the contrary be shifted further out of resonance.

The net force for an atom moving towards $+Oz$ will be directed along $-Oz$, i.e. opposite to its velocity. In the same way, an atoms moving towards $-Oz$ will experience a force along $+Oz$. By summing the two velocity dependent forces one gets with $\vec{k}_1 = -\vec{k}_2 = k\vec{e}_z$:

$$\vec{f}_{tot}(v_z) = \left(\hbar \vec{k}_1 \frac{\Gamma I_{inc}}{2 I_{sat}} \frac{\Gamma^2/4}{(\delta - k_{1,z} v_z)^2 + \Gamma^2/4} + \hbar \vec{k}_2 \frac{\Gamma I_{inc}}{2 I_{sat}} \frac{\Gamma^2/4}{(\delta - k_{2,z} v_z)^2 + \Gamma^2/4} \right)$$

$$\vec{f}_{tot}(v_z) = \hbar \vec{k}_1 \frac{\Gamma I_{inc}}{2 I_{sat}} \left(\frac{\Gamma^2/4}{(\delta - kv_z)^2 + \Gamma^2/4} + \frac{\Gamma^2/4}{(\delta + kv_z)^2 + \Gamma^2/4} \right) \quad (11)$$

For small velocities one can get a linearized expression of this force

$$f_z(v_z) = -\gamma m v_z$$

with a friction coefficient γ :

$$\gamma = -4\delta \frac{\hbar k^2 \Gamma I_{inc}}{M 2 I_{sat}} \frac{\Gamma^2/4}{(\delta^2 + \Gamma^2/4)^2}$$

The velocities of the atoms around $v_z = 0$ will thus decrease exponentially

$$v_z(t) = v_z(t_0) \exp[-\gamma(t - t_0)]$$

The friction coefficient γ is maximum for $\delta = -\frac{\Gamma}{2\sqrt{3}}$. For a laser with an intensity of $I = I_{sat} = 1.6 \text{ mW/cm}^2$:

$$\gamma_{\max} = \frac{9}{4\sqrt{3}} \frac{\hbar k^2}{M} = 25 \mu\text{s}$$

This is a very fast processes. Because of this friction this type of cooling has been called "optical molasses". If one would change the frequency of the laser to positive ("blue") detuning, one would get a heating process for the atoms (increasing their velocities).

The limit of such a cooling process is given by the fluctuations of the forces. These fluctuations are of quantum nature and depend on the recoil velocity $\hbar k$. At each cycle of fluorescence a photon is emitted in a random direction. This yields a random walk in momentum space with a step of size $\delta p = \hbar k$. One thus gets an increase in the kinetic energy of the atomic distribution:

$$\frac{d(\Delta p)^2}{dt} = 2D$$

The diffusion coefficient D is given by the ratio of the step size δp and the time scale for a fluorescence cycle τ :

$$D = \frac{(\hbar k)^2}{\tau}$$

The average time τ between two spontaneous emissions is given by:

$$\frac{1}{\tau} = \frac{\Gamma I_{inc}}{2 I_{sat}} \frac{\Gamma^2/4}{\delta^2 + \Gamma^2/4} \quad (12)$$

This diffusion gives rise to an increase in the kinetic energy, i.e. a heating of the atoms.

At equilibrium the heating due to the fluctuations and the cooling due to the friction effect compensate:

$$\left[\frac{d(\Delta p)^2}{dt} \right]_{\text{eq}} = -2\gamma (\Delta p)^2_{\text{eq}} + 2D = 0 \quad (13)$$

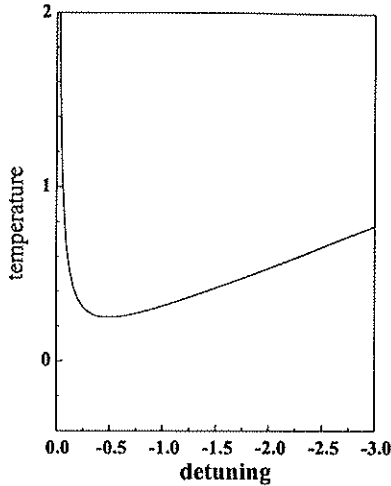


Figure 4. Equilibrium temperature $\frac{k_B T}{\hbar \Gamma}$ as a function of detuning δ/Γ

and one obtains the temperature (figure 1.5):

$$\frac{1}{2} k_B T = \frac{(\Delta p)_{\text{eq}}^2}{2M} = -\hbar \Gamma \frac{(\delta^2 + \Gamma^2/4)}{8\delta\Gamma} \quad (14)$$

The minimum temperature is hence:

$$k_B T_{\text{min}} = \frac{\hbar \Gamma}{4}$$

which is obtained for $\delta = -\frac{\Gamma}{2}$.

Remark A precise calculation needs to take into account the fluctuations of the number of fluorescence cycles and not only the random direction of the spontaneous emitted photons and the three dimensional aspect of the random walk, slightly changed the numerical value of the above result, yielding in a one dimension configuration a so-called Doppler limit of

$$k_B T_{\text{Dopp}}^{(1D)} = \frac{\hbar \Gamma}{3}$$

and in a standard three dimensional situation a Doppler temperature of

$$k_B T_{Dopp} = \frac{\hbar\Gamma}{2}$$

For Rubidium atoms this Doppler temperature is

$$T_{Dopp}^{Rb} = \frac{1}{k_B} \frac{\hbar\Gamma}{2} = 120\mu K$$

□

These are extremely low temperatures, below what has been obtained by other techniques before. In addition, experiments by W.D. Phillips and coworkers in 1988⁷ have resulted in even lower temperatures. In order to explain this lower temperatures one needs to include the spatially modulated dipole forces in the standing wave together with polarization effects and the more complex internal structure of the atoms (optical pumping)^{8,9}. But we will not discuss here such "Sisyphus" cooling¹⁰ or other elegant sub-recoil cooling techniques^{11,8,9}.

2 Interferences in multiple scattering

In the preceding section we have studied the effect of atom-laser interaction in respect of its effect on the atomic momentum and position distribution. This is the main purpose of the community of laser manipulation of atoms. When considering atoms as scatterers, one will on the other side be more interested in the influence of the light scattered by atoms. We want to study in this section to what extent atoms are different from standard scatterers such as TiO_2 or other Rayleigh and Mie scatterers.

2.1 Scattering cross section of single atoms

Atoms can be considered as very strong scatterers with a cross section as large as $\sigma_{at}^{res} = \frac{3\lambda_{at}^2}{2\pi}$ (eq.9). This cross section has a very sharp frequency dependance and is maximum if the laser frequency is tuned to an atomic resonance frequency ($\delta = 0$). The frequency dependance for $\omega \rightarrow 0$ can be deduced from eq. (3): $\sigma = \alpha'' k^{12}$. By replacing Γ with $\Gamma \frac{\omega^2}{\omega_{at}^2}$, one recovers the known ω^4 dependance for Rayleigh scatterers at low frequencies. As the frequency increases one usually gets into the Mie regime with more or less sharp resonances and a cross section which tends to twice the geometrical cross section:

$$\sigma \xrightarrow{\omega \rightarrow \infty} 2\pi r^2$$

Remark The factor of 2 compared to the geometrical cross section can be understood as being the diffracted light ($\theta_{diff} \sim \frac{\lambda}{r}$) by the sphere, which reduces the intensity in the forward beam. This result is valid if one is in the far field ($L \gg \frac{r^2}{\lambda}$) limit. When looking at the cross section of an object by measuring the intensity profile after a short distance ($L \lesssim \frac{r^2}{\lambda}$) the diffracted light is not yet separated from the geometrical image of the sphere and one only recovers the geometrical cross section of πr^2 . \square

From the scattering cross section one defines a mean free path l_{MFP} as the average distance between two successive scattering events:

$$l_{MFP} = \frac{1}{n\sigma}$$

where n is the density of scatterers in the medium.

In this respect one can consider the atomic resonance frequency as a first sharp resonance of the scattering cross section. In real atoms there are many transitions corresponding to different resonances in the cross section. An atomic spectrum is thus equivalent to a frequency dependant cross section. In these terms the finesse of a resonance in the case of atoms is extremely large. For Rubidium atoms e.g. one has:

$$\frac{\omega}{\delta\omega} = \frac{\omega}{\Gamma}$$

With $\omega = \frac{2\pi c}{\lambda}$ one gets for $\lambda = 780nm$ and with a natural linewidth of $\Gamma = 6MHz$ a finesse of

$$\frac{\omega}{\delta\omega} \simeq 6 \cdot 10^7$$

Such large values have been obtained in other types of scatterers^{13,14} but in order to make use of this large values of the finesse in a multiple scattering experiment one needs to be able to produce many scatterers with the same resonance frequency (with a precision of the width of the resonance). This seems to be unrealistic with scatterers such as microspheres. Atoms of the same element however have all exactly the same resonant frequency and are in this respect an unique way of studying high finesse cross section in multiple scattering¹⁵.

If one does not only want to study an extinction cross section (attenuation of the incident beam) it is also of interest to analyze the differential scattering

cross section and the absorption cross section. Let us only consider the far field radiation by a dipole excited by an incident electric field of a linear polarized laser $E_0 \vec{e}_x \exp(-i\omega_L t)$. Using the results of section 1.1, one obtains a dipole \vec{d} :

$$\vec{d} = \varepsilon_0 (\alpha' + i\alpha'') E_0 \vec{e}_x \exp(-i\omega_L t)$$

The scattered field by a dipole is given by:

$$\vec{E}_{scat} = \frac{1}{4\pi\varepsilon_0} \frac{\exp(ikr)}{kr} k^3 \times \left\{ \left(1 + \frac{i}{kr} - \frac{1}{(kr)^2} \right) \vec{d} + \left(-1 - \frac{3i}{kr} + \frac{3}{(kr)^2} \right) (\vec{e}_r \cdot \vec{d}) \vec{e}_r \right\}$$

which in the far field limit gives:

$$\vec{E}_{scat} = \frac{1}{4\pi\varepsilon_0} \frac{\exp(ikr)}{kr} k^3 \left\{ \vec{d} - (\vec{e}_r \cdot \vec{d}) \vec{e}_r \right\}$$

or

$$\vec{E}_{scat} = \frac{1}{4\pi\varepsilon_0} \frac{\exp(ikr)}{kr} k^3 \left\{ (\vec{e}_r \times \vec{d}) \times \vec{e}_r \right\}$$

The scattered field will not always be in phase with the incident field, because of the complex polarizability α of the dipole. The forward scattered field interferes with the incident field. In this direction an imaginary polarizability (on resonance $\alpha = \alpha''$) will correspond to a destructive interference as the dipole is then in quadrature phase compared to the incident field. For a collection of scatterers this can be compared to a complex part of the index of refraction and to an attenuation of the incident beam¹². In any other direction however there will be no interference with the incident beam and only the scattered field has to be considered, with a frequency dependant phase shift.

2.2 Multiple scattering samples in atomic physics

Atom vapors can be used for multiple scattering experiments and both for room temperature and laser cooled samples an optical thickness larger than one can be obtained. One has to pay care to the various situations one can produce in atomic vapors. As examples let us consider 3 accessible situations.

First, an oven of sodium atoms¹⁶ (actually many other elements can be used) at a temperature of $\simeq 200^\circ$ Celsius can yield a density of 10^{13} atoms/cm and an with a sample thickness of $L = 1$ cm on-resonant optical thickness of several 10^3 . In this case however the Doppler shifts of the atoms leads to an inhomogeneous broadening of the cross section. For a detuning of

1GHz however, the Doppler broadening can be neglected. The optical thickness is reduced but remains still larger than unity. In this situation one might have to consider collective effects, such as mean field and its quantum correction^{15,17,18,19} which will modify the multiple scattering properties.

A magneto-optical trap of 10^{10} Rubidium atoms in a volume at a few $100\mu K$ can be obtained in a volume of a few mm^3 . An optical thickness of several 10 has been obtained in several experiments, in particular those who aim at reaching Bose-Einstein condensation of cold atoms. These samples have the advantage of negligible Doppler effect. If the cooling and trapping light is however present, then the multiple scattering leads to a repulsion between the atoms inducing a correlation between the position of scatterers.

A Bose-Einstein condensate of cold atoms^{20,21} has considerable higher densities of atoms, of the order of a few 10^{14} atoms/cc. In this case however the sample does not have the same local fluctuations in the dielectric constant as all atoms are in the same quantum state. This is an extreme case of dependant scattering and transparency and recently superradiance effects have been reported in this situation²². Optical excitation of such condensates is subject to many recent theoretical investigations²¹.

2.3 Dwell time

Another aspect of scattering by an atom with an internal resonance is that there will be a frequency dependant phase shift of the scattered light. This effect can also be put in the time domain as a retardation effect. Different formulations of such retardation times are being used in the community of multiple scattering, such as Wigner time, delay time^{15,23,24,25,26}. We will use a simple delay time interpretation by defining the dwell time as:

$$\tau_D = \frac{\partial \varphi(\omega)}{\partial \omega}$$

For a slab of glass with an index of refraction of $n = 1.5$ e.g. the transmitted light through a thickness L is phase shifted by:

$$\exp(i\varphi(\omega)) = \exp\left(i\frac{\omega}{c}nL\right)$$

In this simple case the dwell time is:

$$(\tau_D)_{glass} = \frac{\partial \varphi(\omega)}{\partial \omega} = \frac{nL}{c}$$

and is n times longer than through free space. In a simple approach this would be the travel time for a light pulse through the sample. Applying this idea to a delay time for light scattering by a damped dipole, we will get with $\alpha = |\alpha| \exp(i\varphi_\alpha(\omega))$:

$$\tan(\varphi_\alpha) = \frac{\alpha''}{\alpha'} = -\frac{\Gamma}{2\delta}$$

a dwell time of:

$$(\tau_D)_{dip} = \frac{2}{\Gamma} \frac{(\Gamma^2/4)}{\delta^2 + (\Gamma^2/4)}$$

For Rubidium atoms e.g. with $\Gamma^{-1} = 25ns$ one has an on resonance ($\delta = 0$) delay time of

$$(\tau_D)_{res} = \frac{2}{\Gamma} = 50ns$$

which would correspond to $15m$ travel distance in free space! When using the radiative transfer equation in multiple scattering, various velocities have to be defined, such as group velocity v_g and transport velocity v_E which can strongly depend on the dwell time^{15,27}. Atomic samples with large dwell times thus seem to be an good testing ground for studying the influence of dwell time in multiple scattering experiments.

2.4 Coherent backscattering of light

When discussing sample parameters for multiple scattering experiments additional aspects have to be studied. One can for example look for a difference between the mean free path l_{MFP} (mean distance between two scattering events) and the transport mean free path l^* (mean distance to loose the initial direction of propagation):

$$l^* = \frac{l_{MFP}}{1 - \langle \cos \theta \rangle}$$

where θ is the angle between the incident and scattered light. Even though the dipole radiation pattern is not isotropic, one has $\langle \cos \theta \rangle = 0$ and hence $l^* = l_{MFP}$.

One particular effect in multiple scattering has been the subject of detailed studies in recent years: coherent backscattering of waves by a random medium. Scattering of wave by a static random medium of scatterers gives rise to a so-called speckle pattern²⁸. Such speckle pattern are observed whether the medium is optically thin, with single scattering being dominant, or optically thick in the multiple scattering regime. When averaging over different

realizations of the scatterer distribution, the scattered intensity will have a smooth angular dependence. The main argument in this explanation is that the detected field is the coherent sum of electric fields with random phases

$$\vec{E} = \sum_j \vec{E}_j \exp(i\varphi_j)$$

The average detected intensity will then be:

$$\langle I \rangle = \left\langle \left| \sum_j \vec{E}_j \exp(i\varphi_j) \right|^2 \right\rangle$$

A first approach will be to suppose the interference terms to average to zero and thus obtain

$$\langle I \rangle = \left\langle \sum_j I_j \right\rangle \quad (15)$$

This argument however is neglecting the particular case of backscattering. Let us group two by two all scattering paths giving a contribution to the detected field, by taking for each path its reverse path (figure 5).

Assuming that the dephasing for the forward and the reverse path are identical²⁹ the phase difference of the two paths will be

$$\Delta\varphi = (\vec{k}_{inc} + \vec{k}_{out}) \cdot (\vec{r}_1 - \vec{r}_N)$$

One can thus see that if the relative position of the scatterers is randomly changing the phase difference is generally also a random parameter and interference terms in eq. (2.4) will be cancelled. However for the particular case of backscattering

$$\vec{k}_{inc} + \vec{k}_{out} = 0$$

the two reverse paths have exactly the same phase shift regardless of the position of the scatterers. Always having such a constructive interference will give rise to an enhanced backscattering peak when averaging over the sample distribution (figure 6).

Remark Note that for a static sample one does not always have a maximum intensity in backward direction. Indeed, even though paths interfere constructively two by two in this direction, the relative phase shift between the various multiple scattering paths ($1 - 2 - \dots - N$ and $1' - 2' - \dots - N'$ for example) do not have a fixed phase relation. \square

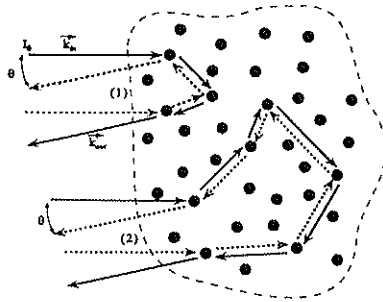


Figure 5. Various contributions to backscattering

This enhanced backscattering relies on the reciprocity of the reverse paths of scattering and on the constructive interference between these two paths. The total width at half maximum of the coherent backscattering cone is given for a half infinite medium by the transport mean free path^{30,31,32,33}:

$$\Delta\theta = 0.7 \frac{1}{kl^*}$$

where k is the wavevector in the scattering medium. This results usually holds for $kl^* \gg 1$. When kl^* become of the order of unity, the so-called Joffe-Régl criterion for strong localization will be obtained^{15,34}. The coherent backscattering cone is a signature of interference effects in multiple scattering. It has been observed with many classical scatterers, but only recently in atomic samples³⁵ (see figure 7).

Albedo Multiple scattering effects have been studied in atom physics^{36,37,17} and radiation trapping and superradiance have been observed. One important aspect in multiple scattering of light by atoms is the frequency spectrum of the scattered light. The radiation spectrum has a complex shape^{38,39,40} and

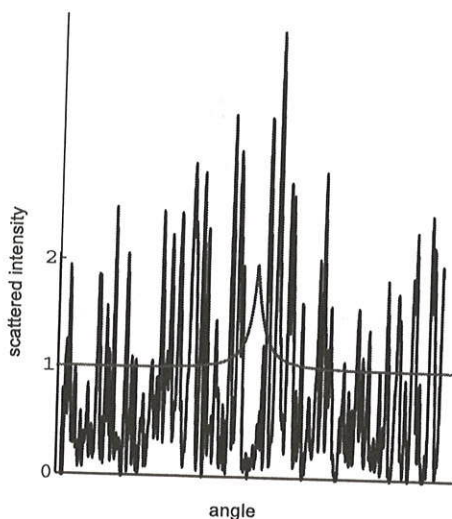


Figure 6. Speckle pattern and averaged intensity

does not only show an elastic component at the drive frequency. In the case of a two level atom an inelastic component appears for larger intensities as the upper state population becomes more and more important and features such as the Mollow triplet have been observed⁴¹. These inelastic components have a spectral width of the order of the natural linewidth of the atomic transition and it seems interesting to investigate what will be the influence of these components on multiple scattering properties such as the coherent backscattering.

For a two level atom the total scattering rate is given by ¹:

$$\Gamma' = \frac{\Gamma}{2} \frac{s}{1+s}$$

where Γ is the width of the excited state and s the saturation parameter:

$$s = \frac{I_{inc}}{I_{sat}} \frac{\Gamma^2/4}{\delta^2 + \Gamma^2/4}$$

This total rate can be separated in an elastic component Γ'_{elas} , having the same frequency spectrum as the incident laser, and an inelastic component

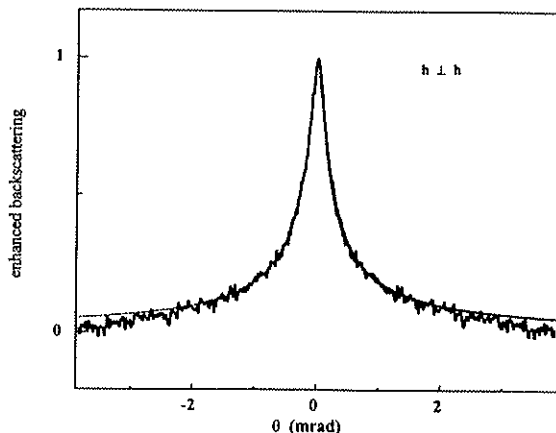


Figure 7. Enhanced backscattering observed from a optically thick laser cooled sample of Rb atoms

Γ'_{inelas} with a broadened spectrum and a triplet structure. One can show that¹

$$\Gamma'_{elas} = \frac{\Gamma}{2} \frac{s}{(1+s)^2}$$

$$\Gamma'_{inelas} = \frac{\Gamma}{2} \frac{s^2}{(1+s)^2}$$

If we suppose that only the elastically scattered light contributes to a coherent backscattering cone (a hypothesis which is under present investigation) one could define an equivalent of an albedo for standard scatterers by:

$$a = \frac{\Gamma'_{elas}}{\Gamma'_{elas} + \Gamma'_{inelas}} = \frac{1}{1+s}$$

which decreases as the transition rate of the atom becomes saturated. Several parameters will be of interest in such a study. Take for example an incident light with a broad spectrum (of $\Delta\nu = 6\text{MHz}$ e.g.). The coherence length for such a laser is of the order of $\Delta x \simeq \frac{c}{\Delta\nu} \simeq 8\text{m}$ which will have to be compared

to the length and time scales of the problem. Another aspect will be the time dependence of the scattered light. In time correlation experiments^{42,43,44} for example this spectral width has to be compared to the detection bandwidth, which varies by orders of magnitude for standard CCD cameras or photomultipliers.

2.5 Strong localization of light in atoms?

One "holy grail" in the multiple scattering community is the observation of strong localization of light. First results have been reported in december 1997 with semi conductor powders⁴⁵. Indeed it is expected that only a high contrast in the index of refraction ($\frac{n_1}{n_2} > 2.5$) would yield strong localization. One problem to obtain this regime can be understood with the following arguments. Consider point scatterers. Let us take the Joffe-Regel criterion for strong localization: $l_{MFP} \sim \frac{\lambda}{2\pi}$. One way to realize this would be to use large wavelength radiation. In this limit the cross section of Rayleigh scattering is scaling as

$$\sigma \underset{\omega \rightarrow 0}{\propto} \omega^4$$

or

$$\frac{\sigma}{\pi R^2} \simeq (kR)^4$$

With the mean free l_{MFP} path scaling as $l_{MFP} \sim \frac{1}{n\sigma} \propto \omega^{-4}$ one obtains for the Joffe-Regel criterion in the low frequency limit ($\omega \rightarrow 0$):

$$kl_{MFP} \propto \omega^{-3} \gg 1$$

However if one uses a resonant scattering, the cross section can be of the order of $\sigma_{res} \sim \lambda^2$ and strong localization might be expected for $n\lambda^3 \sim 1$. In this case however collective radiation effects might become too important to be neglected and could yield a larger mean free path than for independent scatterers. Today it is possible to obtain sample with $n\lambda^3 > 1$ in Bose-Einstein condensates cold atoms, but localization effects in such samples have not yet been observed.

One consequence of atoms closer than one wavelength is the correlation arising from recurrent scattering, which can also be interpreted as dipole-dipole Van der Waals effects. For dilute sample, such that $kl_{MFP} \gg 1$ the correlation can lead to an increased "effective" mean free path (transport mean free path), as the radiation pattern can be affected to give enhanced forward scattering (superradiance e.g.). In dense media, the polarizability is modified

due to local field effect (Lorentz-Lorenz formula^{46,47}) and we have

$$\chi = \frac{n\alpha}{1 - \frac{1}{3}n\alpha}$$

or

$$\frac{\varepsilon - 1}{\varepsilon + 2} = \frac{1}{3}n\alpha$$

with $\varepsilon = 1 + 4\pi\chi$.

Using

$$\alpha = \frac{1}{\delta - i\frac{\Gamma}{2}} \frac{\alpha_0}{2} \omega_L = \frac{\delta + i\frac{\Gamma}{2}}{\delta^2 + \frac{\Gamma^2}{4}} \frac{\alpha_0}{2} \omega_L = \alpha' + i\alpha''$$

one has

$$\varepsilon = 1 + 4\pi n \frac{\frac{\alpha_0}{2} \omega_L}{\delta - i\frac{\Gamma}{2} - \frac{n}{3} \frac{\alpha_0}{2} \omega_L} = 1 + 4\pi n \frac{\delta - \frac{n}{3} \frac{\alpha_0}{2} \omega_L + i\frac{\Gamma}{2}}{(\delta - \frac{n}{3} \frac{\alpha_0}{2} \omega_L)^2 + \frac{\Gamma^2}{4}} \frac{\alpha_0}{2} \omega_L$$

which can be seen as Lorentz-Lorenz shift $\Delta\omega_{LL}$ of

$$\Delta\omega_{LL} = -\frac{n}{3} \frac{\alpha_0}{2} \omega_L$$

or using $\frac{3\lambda^2}{2\pi} = \alpha_0 k \frac{\omega_{at}}{\Gamma}$ or $\alpha_0 \omega_{at} = \frac{3\lambda^3}{4\pi^2} \Gamma$ one has

$$\Delta\omega_{LL} = -\frac{n\lambda^3}{8\pi^2} \Gamma \quad (16)$$

Using the relation

$$\Gamma = \frac{1}{3\pi\varepsilon_0} \frac{\omega^3}{\hbar c^3} \|d\|^2$$

one can rewrite this shift

$$\hbar\Delta\omega_{LL} = -n \frac{\|d\|^2}{3\varepsilon_0}$$

This red-shift of the resonance is thus expected to be small for dilute samples, whereas for BEC samples e.g. one could expect shifts several times the natural line width of the transition.

One can also write this result as:

$$\varepsilon = 1 + 4\pi n \frac{\delta - \frac{n}{3} \frac{\alpha_0}{2} \omega_L + i\frac{\Gamma}{2}}{(\delta - \frac{n}{3} \frac{\alpha_0}{2} \omega_L)^2 + \frac{\Gamma^2}{4}} \frac{\alpha_0}{2} \omega_L = 1 + 4\pi n \frac{\delta + \Delta\omega_{LL} + i\frac{\Gamma}{2}}{(\delta + \Delta\omega_{LL})^2 + \frac{\Gamma^2}{4}} \frac{\alpha_0}{2} \omega_L$$

At resonance, for $\delta = 0$, one thus has

$$\varepsilon = 1 + 4\pi n \frac{\Delta\omega_{LL} + i\frac{\Gamma}{2}}{(\Delta\omega_{LL})^2 + \frac{\Gamma^2}{4}} \frac{\alpha_0}{2} \omega_L$$

which, for the absorptive part, is

$$\varepsilon'' = \frac{4\pi n}{\frac{\Gamma}{2}} \frac{\frac{\Gamma^2}{4}}{(\Delta\omega_{LL})^2 + \frac{\Gamma^2}{4}} \frac{\alpha_0}{2} \omega_L$$

and hence $\frac{\frac{\Gamma^2}{4}}{(\Delta\omega_{LL})^2 + \frac{\Gamma^2}{4}}$ times smaller than without local field effects. This reduction factor

$$\frac{\frac{\Gamma^2}{4}}{(\Delta\omega_{LL})^2 + \frac{\Gamma^2}{4}} = \frac{1}{\left(\frac{n\lambda^3}{4\pi^2}\right)^2 + 1} \quad (17)$$

is equivalent to a reduced resonant cross section:

$$\tilde{\sigma}_{res} = \sigma_{res} \frac{1}{\left(\frac{n\lambda^3}{4\pi^2}\right)^2 + 1}$$

The "on-resonant" mean free path thus is increased:

$$\tilde{l}_{res} = l_{res} \left(\left(\frac{n\lambda^3}{4\pi^2} \right)^2 + 1 \right) = \frac{\left(\frac{n\lambda^3}{4\pi^2} \right)^2 + 1}{n \frac{3\lambda^2}{2\pi}}$$

and one obtains the Joffe-Regel criterion (figure 8)

$$k\tilde{l}_{res} = \frac{\left(\frac{n\lambda^3}{4\pi^2} \right)^2 + 1}{n \frac{3\lambda^3}{4\pi^2}} = \frac{1}{3} \frac{\left(\frac{n\lambda^3}{4\pi^2} \right)^2 + 1}{\left(\frac{n\lambda^3}{4\pi^2} \right)}$$

which is minimal for $\frac{n\lambda^3}{4\pi^2} = 1$ and is then

$$k\tilde{l} \simeq \frac{2}{3} \lesssim 1$$

This model seems to predict that strong localization of light in dense cold atomic vapors can only be obtained for on-resonant excitation in a narrow window of density. One has however to investigate more precisely how the fluctuations of such a "mean field effect" will modify not only the position of the resonance but also its width.

The bare shift of the resonance could be taken care of by taking e.g.:

$$\delta + \Delta\omega_{LL} = 0$$

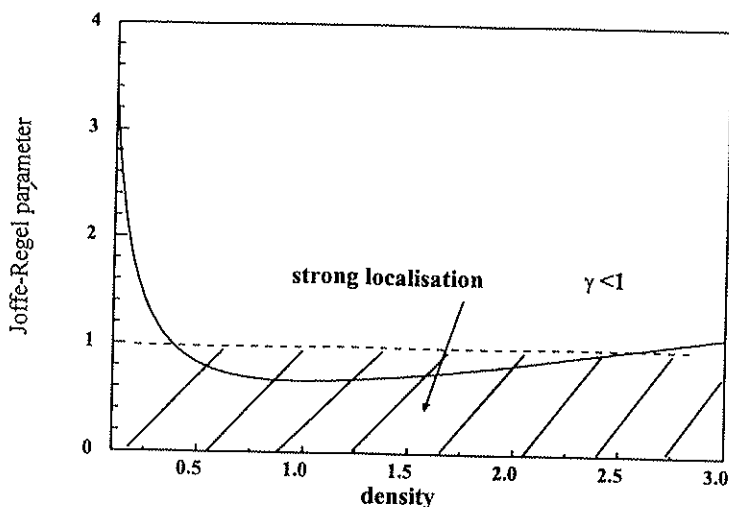


Figure 8. Joffe-Regel parameter γ as a function of density $n\lambda^3/4\pi^2$

One then has:

$$\tilde{l} = \widetilde{l_{res}} \left(\left(\frac{2\delta}{\Gamma} - \frac{n\lambda^3}{4\pi^2} \right)^2 + 1 \right)$$

and hence the detuning dependant Joffe-Regel parameter (figure 9):

$$k\tilde{l} = \frac{1}{3} \frac{\left(\frac{2\delta}{\Gamma} - \frac{n\lambda^3}{4\pi^2} \right)^2 + 1}{\left(\frac{n\lambda^3}{4\pi^2} \right)}$$

This should compensate for this Lorentz-Lorenz shift and it should then be possible to keep the same cross section as without local field effects. In that case the Joffe-Regel criterion seems to be fulfilled when

$$[k\tilde{l}]_{\min} = \frac{1}{3} \frac{1}{\frac{n\lambda^3}{4\pi^2}} < 1$$

i.e. when

$$n\lambda^3 > \frac{4\pi^2}{3} \quad (18)$$

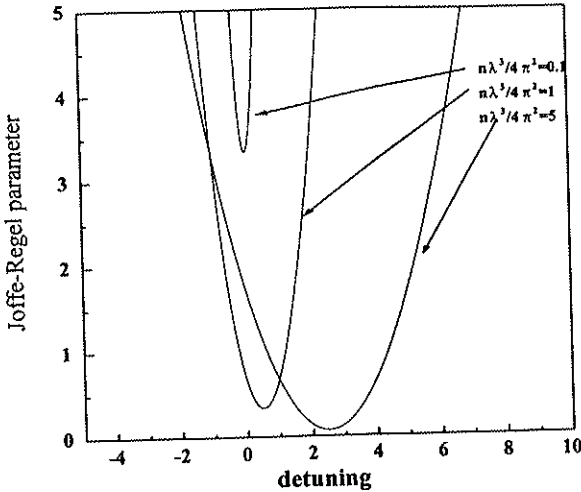


Figure 9. Joffe-Regel parameter γ as a function of detuning δ/Γ

Note that in this result, only the optical wavelength is relevant (as opposed to the threshold for Bose-Einstein condensation where one requires $n\lambda_{dB}^3 \gtrsim 2.613$ and where the De Broglie wavelength is the important parameter)

The Lorentz-Lorenz correction has been described in more detail for optical thin media by Friedberg et al⁴⁶. The precise shift depends on geometrical configuration and give different numerical factors. It is interesting to notice that even the anti-resonant terms (usually neglected in the rotating wave approximation) contributes significantly to the shift. Furthermore collisional shift of the resonance cannot be neglected as it also varies linearly in density (as is well known in atomic clocks):

$$\Delta\omega_{coll} = \beta n$$

This effect is difficult to distinguish from the Lorentz-Lorenz correction as both scale as the density of the atoms. However in a non linear experiment with hot atoms it has been shown that the Lorentz-Lorenz correction has to be taken into account for a precise evaluation of the shift of the atomic resonance in dense media. Furthermore the Doppler effect results in an inhomogeneous broadening of the line and the "effective" average cross section for resonant

detuning will be reduced.

As the temperature dependance of the atoms does not appear explicitly in eq.(18) it is important to note that in addition to the shift of the resonance collisional broadening of the line, which also scales as the density of the atomic gas, cannot be neglected.

3 Conclusion

In this paper we have described the basic cooling mechanism of atoms by laser using a (almost) complete classical description. This approach can be very usefull to estimate the cooling (in-)efficiency in other situations (dielectric spheres with internal resonances e.g.). In the second part we have presented a well known feature of interference effect in multiple scattering which we have applied to atoms as scatterer. Extended basic criteria to high density, we have addressed the question of how the Joffe-Regel criterion is modified and where one can expect strong localization of light in dense atomic samples. This very interesting regime however needs more thorough investigation. We think that multiple scattering in dense cold atomic media is a very promising topic which is now accessible with e.g. Bose-Einstein condensates.

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